THE INFLUENCE OF A SITUATED APPRENTICESHIP MODEL OF PROFESSIONAL DEVELOPMENT ON HIGH SCHOOL MATHEMATICS EDUCATORS' KNOWLEDGE, BELIEFS, AND USE OF REFORM-MINDED INSTRUCTIONAL PRACTICES

by
Paula L. Clark

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Abstract

Mathematics educators' pedagogical decisions, impacted by educator knowledge and beliefs, are perhaps once of the most influential factors contributing to high school students' understanding of and performance in mathematics. Research indicates that student outcomes in mathematics improve in classrooms where students experience reform-minded mathematics instruction. A sociocultural perspective of learning, for students and adults, promotes pedagogy that is grounded in collaboration and discourse. This study, employing a convergent mixed-methods research design, examined changes in high school mathematics teachers' beliefs, knowledge, and implementation of reform-minded mathematics instruction in response to a situated apprenticeship professional development intervention. Informed by needs assessment data suggesting that mathematics teachers favor traditional forms of mathematics instruction in place of reform-minded mathematics instruction, nine high school mathematics teachers participated in a professional development program that was situated in the participants' context, involved active engagement, offered instructional support, and provided opportunities for collaboration and reflection. Significant changes in participants' beliefs regarding reform-minded mathematics instruction and implementation of reform-minded instructional practices occurred. Study results also indicate that participants found value in the collaborative and supportive elements of the situated apprenticeship professional development program. Study implications include informing future professional development programing.

Keywords: mathematics achievement, sociocultural theory, traditional mathematics instruction, reform-minded mathematics instruction, effective professional development

Primary reader: Dr. Karen Karp; *Secondary readers:* Dr. Carey Borkoski, Dr. Dan Meyer

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The student has made all necessary revisions, and we have read, and approve this dissertation for submission to the Johns Hopkins Sheridan Libraries as partial fulfillment of the requirements for the Doctor of Education degree.

Dr. Karen Karp	Karen Karp	November 8, 2019
Adviser	Signature	Date
Dr. Carey Borkoski	Carey Borkoski	11/8/2019
Committee Member	Signature	Date
Dr. Dan Meyer		11/6/19
Committee Member	Signature	Date
Committee Member	Signature	Date

Dedication

This dissertation is dedicated to Timmy, a young man who has ignited my passion for learning. Our work over the past 18 years has driven me to find creative ways to reach my students and has served as the inspiration for my research.

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This dissertation represents the culmination of my development as an educator, student, and scholar-practitioner over the past three years. True to its sociocultural theoretical roots, my development was impacted by the interactions, experiences, and discourse I shared with my doctoral adviser, my dissertation committee, the Johns Hopkins University School of Education faculty, my classmates in the 2016 cohort, and my family and friends.

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Executive Summary

College and career readiness depend on the acquisition of strong mathematics skills, particularly in algebra (National Mathematics Advisory Panel, 2008; Rakes, Valentine, McGatha, & Ronau, 2010). High school and college graduates of the 21st century encounter occupations that rely on strong mathematical reasoning and problem-solving skills (Erdogan & Stuessy, 2015). However, United States students' abilities to compete in a global workforce has been declining (Jang, 2016; Soulé & Warrick, 2015). Students in the United States trail their international peers in mathematics achievement (OECD, 2016; National Center for Educational Statistics, 2015) and pursue post-secondary degrees involving mathematics at much lower rates (National Science Board, 2007). In spite of recommendations to reform mathematics education to better prepare students for 21st century careers (National Research Council, 2008), low levels of achievement in mathematics persist. In fact, data from the National Assessment of Educational Progress reveals that only 25% of 12th-grade students in the United States are performing at grade level in mathematics (National Center for Educational Statistics, 2015).

Factors Contributing to Mathematics Achievement

Ecological systems theory (Bronfenbrenner, 1994) and sociocultural theory (Vygotsky, 1978) provide appropriate theoretical frameworks for understanding the layers of factors that impact the mathematical development of high school students. Bronfenbrenner's (1994) ecological model of development proposes that human development occurs as a result of reciprocal interactions between an individual and persons, objects, and symbols in their immediate and remote environments. In a similar way, sociocultural theory views learning as a developmental process where individual growth depends on interactions with others (Vygotsky, 1978). A literature review of the prominent factors that contribute to low levels of achievement in mathematics for high school students revealed that socioeconomic status (Finn et al., 2016; Sheldon & Epstein, 2005) and executive function skills (Raghubar, Barnes, & Hecht, 2010) along with curricular choices (Boaler & Staples, 2008; Burris, Wiley, Welner, & Murphy, 2008) and education policies (The National Research Council, 1989), impact a student's performance in mathematics. Performance in high school level mathematics courses also depends on earlier levels of achievement in mathematics (Watts, Duncan, Siegler, & Davis-Kean, 2014), mathematics self-efficacy (Williams & Williams, 2010), and language ability (Alt, Arizmendi, & Beal, 2014). Evidence suggests, however, that the classroom teacher and the teachers' choice of pedagogy, has perhaps the most substantial influence on student achievement in mathematics (Coleman & Department of Health USA, 1966; Entwisle & Alexander, 1992; Hanushek & Rivkin, 2010). The importance of teacher effectiveness on student outcomes (Entwisle & Alexander, 1992; Hanushek, 2016; Hanushek & Rivkin, 2012) warranted an examination of mesosystem level factors contributing to educator decisions regarding mathematics pedagogy.

Mathematics Pedagogy

Mathematics educators' choice of pedagogy culminates from a complex array of factors, indicative of the reciprocal relationship between and among system factors (Bronfenbrenner, 1994; Clarke & Hollingsworth, 2002). Pedagogical decisions reflect educator content knowledge (Campbell et al., 2014; Hill, Rowan, & Ball, 2005), pedagogical content knowledge (Baumert et al., 2010), beliefs regarding instructional practices (Avalos, 2011; Guskey, 2002), and expectations that mathematics teachers hold for all students (Boaler & Staples, 2008; Powell, Fuchs, & Fuchs, 2013). The interconnected model of professional growth (Clarke & Hollinsworth, 2002) illustrates the complexity involved in pedagogical decisions, linking implemented practices to educator knowledge, beliefs, experience, and student outcomes.

Mathematics reform guidelines emphasize the need for educators to shift from traditional teacher-centered instruction toward reformed, student-centered instruction that upholds rigorous standards and focuses on knowledge of algebraic concepts, the ability to reason, and the ability to problem-solve (National Council of Teachers of Mathematics [NCTM], 2014; Rakes et al., 2010). Despite these guidelines, mathematics instruction continues to lack rigor, a conceptual focus, and a student-centered approach and fails to meet the needs of all learners (Boaler & Sengupta-Irving, 2015; Jong, Pedulla, Reagan, Salomon-Fernandez, & Cochran-Smith, 2010). Discrepancies between recommended and implemented instructional practices affect student achievement in mathematics. Given the impact that mathematics pedagogical decisions have on student achievement in mathematics a needs assessment was conducted to understand the context, problem, and needs of a Midwestern public school district.

Factors Influencing Local Levels of Achievement

The problem of practice is set in a Midwestern, suburban, PreK-12 district serving over 7,750 students. Enrollment data revealed that approximately 2,484 students attend high school in this district where four years of mathematics is a graduation requirement. Mathematics achievement is a growing concern for the stakeholders of this district due to recent and historically low performance on state-end-of course exams (Clark, 2017). The needs assessment focused on exploring the demographics, mathematics beliefs, and instructional practices of high school algebra teachers (n=10) in the participant district. Participants completed a questionnaire in which they provided demographic data and self-reported personal mathematics beliefs and instructional practices. Classroom observations were conducted using the Reformed Teaching Observation Protocol (RTOP) (Sawada et al., 2002) which characterizes instructional practices on a scale (0-100) from traditional (0-30) to reform-minded (60-100). RTOP data illustrate that the algebra teachers in this district employed teacher-centered instruction, such as lecture and note-taking, as a predominant form of instruction (Clark, 2017). Needs assessment findings led to the identification of instructional practices as a factor influencing mathematics achievement for the students of this district.

Intervening to Improved Mathematics Achievement

A strong conceptual and procedural understanding of mathematics contributes to the development of number sense (Dehaene, 2011) and numerical cognition (Rittle-Johnson & Schneider, 2015). Interventions that strengthen a student's ability to self-explain mathematics procedures and concepts (Chi, 2000; Siegler et al., 2012), analyze correct and incorrect worked examples (Booth et al., 2015; Lynch & Star, 2013), and create multiple representations when problem-solving (Larbi & Mavis, 2016; Satsangi, Bouck, Taber-Doughty, Bofferding, & Roberts (2016) have been shown to improve student mathematical knowledge. Aside from specific skill or procedural interventions, instructional effectiveness remains a key factor influencing student mathematics knowledge and achievement in mathematics (Campbell et al., 2014; Coleman & Department of Health USA, 1966; Hanushek & Rivkin, 2010; Wenglinsky, 2002). Mathematics teachers that employ reform-minded instructional practices positively impact student learning Eddy et al., 2015; Holt et al., 2015; NCTM, 2014).

Support for Reformed Mathematics Instruction

Mathematics pedagogical approaches impact student learning at varying rates (Boaler & Staples, 2008; Wenglinsky, 2002; Witzel, Mercer, & Miller, 2003), yet empirical literature establishes a clear connection between reform-minded instructional practices and student achievement in mathematics (Boaler & Staples, 2008; Jong et al., 2010; Holt, Young, Keetch, Larsen, & Mollner, 2015; Piburn & Sawada, 2002). Reform-minded mathematics instruction supports students as they develop a conceptual understanding mathematics through authentic, collaborative, and exploratory tasks (Eddy et al., 2015; Sawada et al., 2002). Reform-minded mathematics instruction positively impacts student growth and performance (Boaler & Staples, 2008; Jong et al., 2010; Holt, Young, Keetch, Larsen, & Mollner, 2015; Piburn & Sawada, 2002). Along with preparing students for success in post-secondary and career opportunities (National Mathematics Advisory Panel, 2008; Rakes et al., 2010), reform-minded instruction increases students' conceptual understanding of mathematics (Lawson, Benford, Bloom, & Carlson, 2002), leads to improved problem-solving and reasoning (Mevarech & Kramarski, 2003), and supports mathematics cognition for students who struggle with mathematics (Myers, Wang, Brownell, & Gagnon, 2015).

Social interactions and discourse form the foundation for the collaboration and engagement required of reform-minded mathematics instruction (Eddy et al., 2015; Sawada et al., 2002), aligning with a sociocultural perspective of learning (Vygotsky, 1978). Sociocultural theorists purport that a student's cognitive development relies on their social interactions with more knowledgeable peers and teachers, and promote the use of discourse as a tool for learning (Vygotsky, 1978). The relationship between dialogue and learning underpins a sociocultural theory (Rohlwing & Spelman, 2014) and NCTM (2014) stresses the importance of this relationship in adopting the promotion of discourse as a foundational mathematics practice. Vygotsky (1978) maintains that social interactions activate physiological processes promoting cognitive development. Research posits that mathematics instruction that promotes social interactions increase mathematics performance for all learners (Lambert & Sugita, 2017; Stein, Engle, Smith, & Hughes, 2008).

Transforming Mathematics Pedagogy

Based on data from the local needs assessment and the extant literature regarding the importance of reform-minded instruction, mathematics teachers at the participant high school would benefit from participation in a professional development program designed to transform current instructional practices. Research confirms that professional learning is essential for instructional reform (Desimone, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Guskey, 2002). Given the prevalence of traditional instruction practiced among these mathematics teachers, the instructional transformation will require significant changes in practice. Professional development that seeks greater change can result in higher levels of implementation fidelity (Anderson, 2017), provided that the professional learning programs are sustained, actively engage participants, and support learners in transforming practice (Darling-Hammond et al., 2017; Desimone & Garet, 2015)

Given the mathematics needs of the students and teachers at the participant high school, a professional development intervention was implemented. The purpose of the intervention was to increase the prevalence of reform-minded instruction for the mathematics teachers at this high school. The proposed intervention includes situated, learner-centered, collaborative, and supportive components reflected in designs of effective professional learning programs (Darling-Hammond et al., 2017; Desimone, 2009, Learning Forward, 2011). A situated apprenticeship model of professional development (Clarke & Hollingsworth, 2002; Prather & Brissenden, 2009), a model where teachers take on the role of learner, sought to transform the knowledge, beliefs, and instructional practices of the mathematics teachers at this high school. Aligning with a sociocultural perspective and the tenets of reform-minded instruction for students, this situated apprenticeship professional development intervention investigates six research questions.

Implementation of the Professional Development Intervention

The purpose of this mixed-methods study was to increase the prevalence of reform-minded instruction for the mathematics teachers at this high school. A situated apprenticeship professional development (SAPD) program, on that embedded learner-centered, collaborative, and supportive activities within the teachers' workday, was designed and implemented. The SAPD program was designed from a sociocultural perspective that views educator professional growth as a developmental process reliant on social interactions and experiences with colleagues (Raphael, Vasquez, Fortune, Gavelek, & Au, 2014). Nine high school mathematics teachers participated in the year-long SAPD program which involved participation in 15 face-to-face sessions and six individual classroom instructional modeling sessions. The SAPD program included both outcome and process evaluations and sought to determine how mathematics teachers' (a) use of reform-minded instructional practices, (b) pedagogical beliefs, and (c) knowledge of reform-minded instructional practices changed as a result of participation in the SAPD program. In addition, the study examined the aspects of the SAPD intervention that mathematics teachers describe as most and least impactful in their transformation toward reform-minded instruction. The study also included an assessment of implementation fidelity, the extent to which the SAPD program was implemented as designed.

Data Collection and Analysis

This mixed-methods study employed a convergent parallel design involving a simultaneous collection of quantitative and qualitative data (Creswell & Plano Clark, 2011). Quantitative analysis involved collecting pre- and post-intervention data on instructional practices, educator beliefs, and knowledge of reform-minded instruction were collected via the Mathematics Teaching Pedagogical and Discourse Beliefs Instrument (Lischka & Garner, 2016), the RTOP (Sawada et al., 2002), and researcher designed SAPD session exit tickets. Participants also completed the Panorama Teacher Survey (Panorama Education, 2015) as a quantitative measure of participant responsiveness to the SAPD program. Qualitative analysis involved reviewing SAPD artifacts and semi-structured interview responses for a priori and emergent codes. The merging of quantitative descriptive and inferential statistics with qualitative themes yielded a cohesive understanding participants' experiences in the SAPD intervention.

Findings

Outcome evaluation results indicated that participants experienced significant changes in (a) beliefs regarding reform-minded mathematics instruction and (b) implementation of reform-minded instructional practices. These findings supported a conceptual framework depicting an interconnected relationship between pedagogical beliefs and instructional practices found in extant research (Clarke & Hollingsworth, 2002; Desimone, 2009; Guskey, 2002). Contradictory to hypothesized expectations, participants' knowledge of reform-minded instruction did not change. Qualitative findings confirmed quantitative results indicating that participants (a) were knowledgeable about the characteristics of reform-minded mathematics instruction, (b) engaged in the critical reflection (Mezirow, 1997) necessary in the transformation of beliefs, and (c) increased their use of reform-minded instructional practices. While qualitative data aligned with a priori themes, concerns regarding the use of reform-minded instruction in high-stakes testing content areas arose.

Process evaluation results indicated that participants valued the sociocultural approach to professional development offered by the SAPD program. Sociocultural theory of adult learning stresses the importance of active engagement, collaboration, and individual support (Raphael et al., 2014). Participants found value in the (a) learner-centered approach, (b) opportunity to collaborate with colleagues, and (c) level of individual support offered through instructional modeling. Administrative decisions to alter meeting times and participant absences from school prevented the SAPD intervention from being implemented as planned. Given the relationship between the duration of professional development and the likelihood of sustained implementation (Desimone, 2009; Saderholm, Ronau, Rakes, Bush, and Mohr-Schroeder, 2017), a fully implemented SAPD program could have achieved different results.

Implications

The results of the SAPD intervention contribute to the literature informing the characteristics of effective educator professional development. The success of the program at inducing changes in participants' beliefs regarding reform-minded instruction and use of reform-minded instructional practices can inform future professional development programming for mathematics teachers in this district and the mathematics education community at large. Statistical conclusion validity can be improved by (a) replicating the study in other high schools, (b) expanding the study to include middle school and elementary school mathematics teachers, and (c) redesigning the study to include a control group. As it stands, this research can serve as a pilot study or an evaluability assessment, advising future programming efforts.

Chapter 1: Introduction to the Problem of Practice

Algebra is a pivotal topic of study in the mathematics instruction of students as they progress from kindergarten through Grade 12. Algebra is the bridge between the concrete mathematical concepts taught in elementary school and the abstract representations used in coursework beyond algebra (Booth et al., 2015; Bush & Karp, 2013). Furthermore, a firm understanding of algebra is necessary for students to be both college and career ready and to be competitive in the job market of the 21st century (Eddy et al., 2015; National Mathematics Advisory Panel, 2008; National Research Council, 1989). The age at which algebraic concepts should be introduced to students is a topic of continuous debate and many researchers believe that an introduction to algebra at the elementary level would provide students the opportunity to internalize algebraic concepts rather than memorize algebraic rules (Carraher, Schliemann, Brizuela, & Earnest, 2006; Stephens, Blanton, Knuth, Isler, Gardiner, 2015). Regardless of this debate, mathematics educators and policy makers agree that algebra is a critical component of a K-12 curriculum and a subject that should be accessible to each and every student (Eddy et al., 2015; National Council of Teachers of Mathematics [NCTM], 2014). A comprehensive understanding of algebraic concepts and applications, through rigorous instructional practices was the recommendation of the National Mathematics Advisory Panel (2008) and the panel's recommendation have become the framework for current algebraic standards (Eddy et al., 2015; NCTM, 2014).

Algebraic concepts, procedures, and reasoning are often challenging for many learners. Results from the 2015 National Assessment of Educational Progress report that only 25% of 12th grade students performed at or above a proficiency level in mathematics. Data from the 2015 Program for International Student Assessment (PISA) places the United States at a below average score of 470 in mathematics (M = 490; N =70). The below average score is also indicative of downward trend based on past PISA scores. Furthermore, for students with specific language impairment (SLI), the challenges of learning algebra are even greater. Specific language impairment occurs when children with typical cognitive development have difficulty learning their native language (Alt, Arizmendi, & Beal, 2014).

Language ability involves such skills as phonological processing and verbal working memory ability. Phonological processing reflects the ability to control individual sounds within words whereas language ability involves storing and retrieving verbal information (Nys, Content,

& Leybaert, 2013). Students with SLI have deficits in these areas of language ability (Nys et al., 2013). Phonological processing and verbal working memory are necessary for number naming, counting, and developing numerical representations (Morin & Franks, 2010; Nys et al., 2013). In addition, the presence of SLI affects early numeracy skills such as counting, number naming, and fact retrieval (Alt et al., 2014; Fazio, 1999; Morin & Franks, 2010). Neurocognitive research confirms the role of language processing in mathematical development (Barnes & Raghubar, 2014). Although SLI often lead to difficulties in arithmetic calculation and reasoning, many educators continue to believe that numeracy and literacy are independent (Donlan, Cowan, Newton, & Llyod, 2007). Regardless of this discrepancy, early mathematics ability can predict high school mathematics achievement, signifying the importance of strong foundational skills.

Teaching algebra to students with complex learning needs requires both strong mathematical content knowledge and pedagogical content knowledge (Baumert et al., 2010; Boyd et al., 2012; Darling-Hammond, 2000). Furthermore, teachers must be knowledgeable about the mathematics curriculum, understanding of how algebraic concepts evolve and rely on one another, adept at anticipating how difficulties and misconceptions may interfere with student understanding (Egodawatte, 2009). Effective pedagogical methods are associated with improved mathematics achievement and the use of reformed teaching practices leads to increased pupil learning (Jong, Pedulla, Reagan, Salomon-Fernandez, & Cochran-Smith, 2010; Sawada et al., 2002). Reform teaching practices follow the principles outlined in the *Principles and Standards for School Mathematics* (NCTM, 2000) and encourage learning that is student led, collaborative, exploratory, engaging, and concept oriented (Piburn & Sawada, 2000). Along with decisions regarding pedagogy, student achievement in mathematics is affected by curriculum offerings (Boaler & Sengupta-Irving, 2015; Burris, Wiley, Welner, & Murphy, 2008; Gamoran, Porter, Smithson, & White, 1997). Rigorous curriculum and high expectations are characteristic of reform minded teaching and benefit the mathematics learning of all students (Boaler & Staples, 2008).

Problem of Practice

College and career readiness depends on the acquisition of strong mathematics skills, particularly in algebra (National Mathematics Advisory Panel, 2008; Rakes, Valentine, McGatha, & Ronau, 2010). A strong command of algebra leads to improved mathematical reasoning and problem solving, skills necessary for success in careers of the 21st century (National Mathematics Advisory Panel, 2008). However, national and international data illustrate low levels of achievement in mathematics for students in the United States (National Center for Educational Statistics, 2015; OECD, 2016;). In fact, data from the National Assessment of Educational Progress reveals that only 25% of 12th-grade students in the United States are performing at grade level in mathematics (National Center for Educational Statistics, 2015). Mathematics reform guidelines emphasize the need for educators to shift from traditional teacher-centered instruction toward reformed, student-centered instruction that focuses on knowledge of algebraic concepts, the ability to reason and problem-solve, and rigorous standards (National Council of Teachers of Mathematics, 2014; Rakes et al., 2010). Despite these guidelines, mathematics instruction continues to lack rigor, a conceptual focus, and a student-centered approach and fails to meet the needs of all learners (Boaler & Sengupta-Irving, 2015; Jong et al., 2010). Decisions regarding mathematics curriculum and instruction (Boaler & Staples, 2008; Burris et al., 2008; Gamoran et al., 1997) as well as teacher pedagogical beliefs (Campbell et al., 2014; Gamoran et al., 1997; Maccini & Gagnon, 2002) influence mathematics achievement for students who have difficulty in mathematics, students with disabilities, and mathematics students in general.

Ecological Systems Theoretical Framework

The factors contributing to students' difficulties with mathematics are complex. Family, student, and educator factors form an intricate array of complex interactions that affect one's ability to understand algebraic concepts. For example, Finn et al. (2016) compared functional MRI imaging and mathematics achievement scores of 67 middle school students from low and high income families. The MRI data were collected while students were completing an *n*-back task, a task which places demands on working memory by having students recall correct pairings of information *n* trials back. The study supports the interrelatedness of factors by concluding that students' low socioeconomic status (SES) is associated with their lower mathematics achievement and less capacity in working memory (Finn et al., 2016). Working memory has also been linked to problem-solving skills, a necessary component of an algebra curriculum (Swanson & Beebe-Frankenberger, 2004). Because SES is related to working memory capacity and brain responses to demands on working memory (Finn et al., 2016), SES is a factor that impacts students' problem-solving skills. The complexity of the interconnectedness between the factors that contribute to algebraic learning, and the relationships between them, is described by ecological systems theory (Bronfenbrenner, 1977).

Ecological systems theory (EST), whether depicted as a nested model (Bronfenbrenner & Morris, 2006) or a networked model (Neal & Neal, 2013), "is widely recognized for underscoring the importance of interdependent and multilevel systems on individual development" (Neal & Neal, 2013, p. 723). Bronfenbrenner (1994) uses the term "proximal processes" (p. 1644) to describe the reciprocal, ongoing interactions between an individual and the surrounding environments. Environmental interactions impact the development of the focal individual. From direct interactions to indirect interactions, these systems are referred to as the microsystem, mesosystem, exosystem, and macrosystem (Bronfenbrenner, 1994).

Factors affecting algebra achievement of high school students can be framed by EST. In this model, no one factor is given more importance than others, but it is the relationship between the factors that affect algebraic development and achievement. Many factors, such as administrative decisions regarding hiring and choosing curriculum and educator factors such as content knowledge, are indirectly related to algebra students and occur at the macrosystem and exosystem levels respectively. Other factors, student ability, teacher pedagogy, and teacher expectations, directly interact with the student and occur at the mesosystem and microsystem levels. A graphical model of the factors within each system is shown in Figure 1.1.

The following literature review is framed within the nested model (Neal & Neal, 2013) of EST. Factors affecting algebra achievement will be discussed from the macrosystem level to the microsystem level. The review of the literature will conclude with a brief discussion of the constructs affecting the professional context of this researcher.

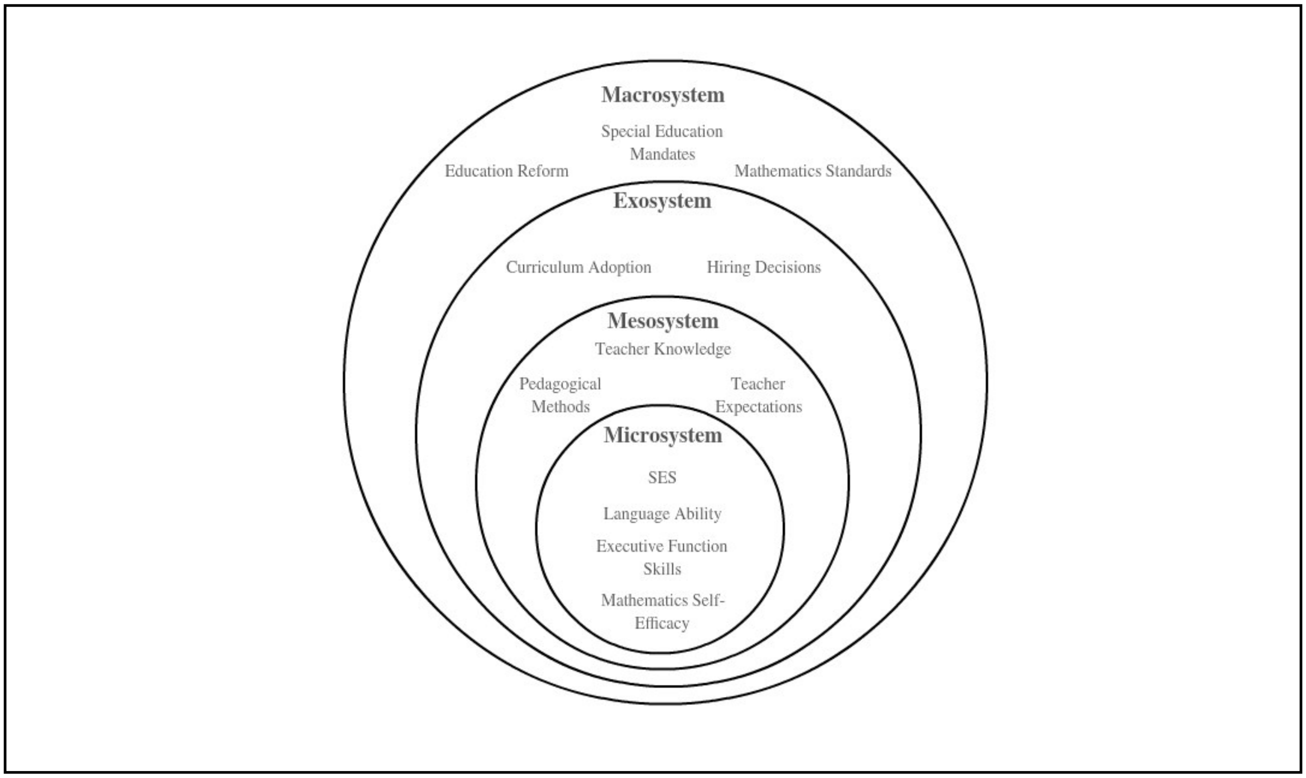

Figure 1. 1. A hierarchy of factors related to algebra achievement presented in a nested model of ecological systems theory (Neal & Neal, 2013).

A Review of the Literature on Factors Influencing Algebra Achievement

An investigation into the factors related to algebra achievement is provided by the following literature review. Findings from the existing literature are synthesized and the constructs are outlined by an EST framework. This review begins at the macrosystem level.

Macrosystem: Mathematics Reform

At the macrosystem level, the attitudes, beliefs, practices and customs of society are studied pertinent to their influence on the development of an individual (Bronfenbrenner, 1994). The education system in the United States has been in continual phases of reform (Carraher et al., 2006; Eddy et al., 2015; Tyack & Cuban, 1995) and these reforms affect the instructional of algebra. The need for educational reform in the United States was spurred by the publication of

A Nation at Risk (Gamoran et al., 1997; National Research Council, 1989; Pugach & Warger, 2001). This report, written by the National Commission on Excellence in Education (Gardner, 1983) outlined the decline of student achievement in America and implored the American people to turn from an acceptance of educational mediocrity. The need for educational reform in the area of mathematics alone was evident from stark findings such as a 40 point decline in average Scholastic Aptitude Test mathematics scores, a 72 percent increase in the need for remedial mathematics course in college, and the inability of most 17 year olds (66%) to solve mathematics problems with more than one step (Gardner, 1983).

Following the release of *A Nation at Risk,* the National Research Council published *Everybody Counts* (1989), a report illustrating the importance of increasing the rigor of mathematics instruction to meet the needs of an ever-changing society. The National Research Council (1989) proposed that all students, those who will further their study of mathematics in college, as well as those who will complete their study of mathematics in high school, have similar quantitative literacy needs. Regardless of the path students take after completion of the secondary education, a strong understanding of algebra will allow students to become college or career ready (Eddy et al., 2015). The ability to reason, communicate, and problem-solve, skills necessary for a firm mathematics foundation, are outlined by NCTM in *Principles and Standards for School Mathematics* (2000) and *Principles to Actions* (2014). Like the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), the NCTM standards provide a unified set of core algebraic skills and best practices that serve as educator guidelines for improving mathematics instruction, and when implemented improve students' conceptualization of mathematics.

Unfortunately for students who struggle to learn mathematics, and specifically for students with disabilities (SWD), mathematics reform efforts failed to address the needs of all learners (Pugach & Warger, 2001). Programs for students with disabilities continue to lack rigor and even deny students access to the general curriculum (Boaler & Sengupta-Irving, 2015; Pugach & Warger, 2001). Tracking students into low-level mathematics courses or college preparatory courses, based on ability, was an answer to accountability issues and proposed to provide students with mathematics instruction aligned to their needs (Burris et al., 2008). However, Burris et al. (2008) report that low expectations, less rigor, and less effective teachers are often characteristic of low-level mathematics courses.

The 1997 Individuals with Disabilities Education Act (IDEA) was accompanied by the use of an Individual Education Program (IEP), provisions that defined how students with disabilities would access the general curriculum (Pugach & Warger, 2001). Inclusive classrooms, those taught by both a content specialist and an intervention specialist, were a response to the demands that all students should have access to the general curriculum (Van Reusen, Shoho, & Barker, 2001). However, placement into general education classes did little to ensure that students with disabilities would be successful at accessing the same curriculum as their non-disabled peers (Pugach & Warger, 2001).

Reform efforts for students with disabilities, including placing students in inclusive classrooms (courses co-taught by mathematics content teachers and special educators) introduced further challenges. A cross-sectional study of 129 mathematics educators revealed common problems of inclusive mathematics classrooms (Maccini & Gagnon, 2002). Mathematics teachers from Kindergarten through Grade 12 participated in the study assessing their perceptions of their familiarity with, and confidence in applying the NCTM standards. Both secondary mathematics content teachers (*n* = 65) and special educators (*n* = 64) answered questionnaires regarding their understanding of NCTM standards. Although 95% of the content educators reported familiarity with the standards, only 55% of the special educators reported familiarity (Maccini & Gagnon, 2002). Besides the lack of familiarity, levels of confidence in applying the NCTM content standards to instruction was significantly lower ($p = .004$) for special education teachers (Maccini & Gagnon, 2002). In response to the statement "I feel confident in my ability to teach mathematics relative to the goals of the NCTM standards" (Maccini & Gagnon, 2002, p. 335) where 1=strongly agree and 5=strongly disagree, the mean rating for content teachers was 1.97 and the mean rating for special education teachers was 2.59. In addition to reporting less confidence, special educators in this study reported taking fewer mathematics methods courses and, except for basic mathematics skills, report unfamiliarity with secondary mathematics topics including pre-algebra and algebra. The mathematical content knowledge and pedagogical knowledge of teachers is positively associated with student achievement (Campbell et al., 2014), therefore a lack of both confidence and familiarity of mathematical standards by special educators can influence student performance in algebra.

Exosystem

The algebra achievement of individual learners is affected by decisions made at the district level. The school district of the algebra learner comprises an exosystem, the next component of ecological systems theory (Bronfenbrenner & Morris, 2006). As is characteristic of exosystem factors, district decisions do not actively involve the algebra learner, yet they greatly influence the ability of the learner to achieve. Decisions regarding curriculum and hiring directly affect the algebra achievement of the students in the district.

Curriculum. Individual school districts are ultimately responsible for choosing the curriculum and defining the learning environments in which the students will be instructed. Mathematics curriculum decisions and placement into mathematics courses greatly influence algebra achievement, particularly for students who struggle in mathematics (Boaler & Staples, 2008; Burris et al., 2008; Gamoran, et al., 1997).

The effects of three mathematics curriculum pathways for both low-achieving and low-income high school students is the focus of a mixed-methods study by Gamoran et al. (1997). The schools were purposefully sampled (*n*=7) to fit the researcher's criteria. Chosen schools scored in the lowest quartile on standardized achievement tests, had high percentages of students on free and reduced lunches, and offered the three designated curricular pathways; general track, transitional, and college-preparatory. General track mathematics courses offered basic mathematics and pre-algebra instruction. Transition courses combined arithmetic, algebraic, and geometric concepts and were taught to students with mathematics difficulties. College-preparatory courses began with traditional algebra 1 instruction. Researchers measured the mathematics achievement of 886 students using a researcher created mathematics assessment based on the 1990 National Assessment of Educational Progress assessment. Participation in general track mathematics courses, transition courses, and college-preparatory courses, was compared to students' mathematics achievement scores. Differences in achievement between transition courses and college-preparatory course were inconclusive, however, findings indicated that general track mathematics courses offer weak instruction and hinder student achievement in mathematics (Gamoran et al., 1997). The transition courses in this study were designed to assist students who lack strong mathematical skills prepare for the rigor of algebra by emphasizing mathematical reasoning and were favored over general track courses for low-achieving high school students (Gamoran et al., 1997).

The choice of curriculum was found to be a contributing factor of a students' ability to obtain a Regents diploma or a Diploma of the International Baccalaureate in New York (Burris et al., 2008). To earn a Regents diploma, along with a local diploma, students must pass Regents exams in several areas, along with completing the requirements for their district. The Diploma of International Baccalaureate (IB) is awarded to students participating in a rigorous course of study, which includes a battery of assessments pertinent to the program. In a longitudinal quasi-experimental study Burris et al. examined the effects of detracking efforts to improve the likelihood of obtaining a Regents diploma or Diploma of IB. Practice Scholastic Assessment Test data, Regent Diploma attainment, and IB data, along with demographic data was obtained from two cohorts of students over the course of three years. The second cohort of students entered high school after efforts were put into place to instruct all students in mathematically accelerated, heterogeneous classrooms, while the first cohort of students served as a control group. The detracked cohort of students entered middle school at a time when all students were taught using the same accelerated mathematics curriculum. In accordance with the main purpose of the study, researchers concluded that students in the detracked cohort increased their odds of attaining a Regents diploma by almost six times as compared with peers of similar aptitude. Furthermore, findings suggested that the positive effects of detracked curriculum are the highest for minority and low SES students (Burris et al., 2008).

The results of these studies are contradicted by a longitudinal observational study conducted after a policy change in the Chicago Public School system requiring all ninth-grade students to take algebra (Allensworth, Nomi, Montgomery, & Lee, 2009). Data on two cohorts of students, those who enter high school prior to the policy implementation and those who enter high school after, were analyzed and although students in all ability groups were more likely to earn algebra credit, the results for subgroups were less promising (Allensworth et al., 2009). For low ability students, failure rates and absenteeism increased while math grades decreased (Allensworth et al., 2009). The authors recognize that factors other than curriculum, such as the quality of instruction, preparedness of the teachers, and instructional methods (Allensworth et al., 2009), need to accompany curricular changes which aligns with the cautions identified by Burris et al. (2008). The district in the study by Burris et al. (2008), as with districts studied by Boaler and Staples (2008) and Lalvani (2013), places great value on reform efforts. This caution indicates the value of the mesosystem factors of teacher expectations and practices which will be discussed in subsequent sections.

Perhaps one of the most convincing arguments for increasing the curricular demands for all students is provided by the research of Boaler and Staples (2008), who purposefully sampled three high schools to compare student achievement, teaching approaches and student perceptions through a mixed-methods design (Boaler & Staples, 2008). As described by Boaler and Staples (2008), school size and teacher composition were similar in the three schools. However, one school, Railside school, a fictitious name, reports greater ethnic and cultural diversity and a higher percentage of low- income students than the two comparison schools, fictitiously named Hilltop and Greendale. An even greater difference between these schools was the curriculum offering. Whereas students at Hilltop and Greendale schools could choose between traditional mathematics courses taught with traditional instructional practices, all students at Railside schools were enrolled in college-preparatory mathematics classes and taught with reform-minded instructional approaches. The mathematics achievement of these students was measured by both content tests and projects (open-ended collaborative assessments), along with scores on the California Standards Test of algebra. Findings indicate that students at Railside, who began this study with lower levels of mathematics achievement, significantly ($p < 0.001$) outperformed their peers from Hilltop and Greendale. These findings were also indicative of the beliefs and reform-minded instructional practices of the teachers at Railside, which will be discussed within the mesosystem layer.

Hiring Implications. The main commonality among research on the positive effects of offering rigorous curriculum to all students in heterogeneous classrooms is the classroom teacher (Boaler & Staples, 2008; Burris et al., 2008; National Mathematics Advisory Panel, 2008; Gamoran, et al., 1997). A common thread among the research is that the content knowledge and beliefs of the teachers are paramount (Campbell et al., 2014; Depaepe, Verschaffel, & Kelchtermans, 2013; National Mathematics Advisory Panel, 2008). Although individual teacher qualities affect the students more at a mesosystem level, the hiring of effective and dedicated teachers is done at the exosystem level, again strengthening EST argument on the interconnectedness of the various systems.

Seeking to empirically define the availability, or subsequent shortage of mathematics and science teachers, Ingersoll and Perda (2010) gathered data from three national surveys, the Schools and Staffing survey, the Teacher Follow-Up Survey, and the Baccalaureate and Beyond Survey. Survey data were collected from the National Center for Educational Statistics. Along with the survey data, information on the supply of new teachers was gathered from the Integrated Postsecondary Educational Data System. The data, collected over the course of one year, were analyzed to address numerous research questions, among which were questions concerning the supply of teachers due to the demands of student enrollment, the supply of mathematics teachers, teacher turnover, and variation in staffing due to school demographics. Ingersoll and Perda (2010) discuss several conclusions potentially effecting algebra students. The researchers recognize that high poverty, high-minority, urban schools have the most difficult time finding and retaining mathematics and science teachers (Ingersoll & Perda, 2010; Darling-Hammond, 2000). Also, Ingersoll and Perda (2010) conclude that hiring difficulties vary by school location, most times even within districts, indicating that other factors affect staffing problems. Although this study provides insight into the difficulties that some districts face in hiring mathematics teachers, a factor which affects students in the algebra classroom, it provides information on the overall supply of available certified mathematics teachers, not the supply of effective mathematics teachers (Ingersoll & Perda, 2010).

The hiring and retention of effective mathematics teachers impacts students in high poverty, high minority districts the most (Darling-Hammond, 2000; Entwisle & Alexander, 1992; Ingersoll & Perda; 2010). The unequal distribution of effective teachers across educational settings is a problem of equity (Darling-Hammond, 2000). Discussion regarding these historical inequities in the distribution of effective teachers needs to take place at the macrosystem level. However, the impact of effective teachers on all students, advantaged and disadvantaged, can be seen at the mesosystem level, in the classroom.

Mesosystem

At the mesosystem level of ecological system theory interactions between the focal individual and others involve direct, face-to-face encounters (Bronfenbrenner, 1977). Interactions in the classroom, between the algebra student, their peers, and their instructors occur at the mesosystem level. Factors such as teachers' mathematical content knowledge, pedagogical content knowledge and expectations regarding their students' ability to learn mathematics directly influence the learning of the focal individual, the algebra student, and therefore affect student achievement. The classroom teacher continues to have the largest influence on student achievement (Campbell et al., 2014; Wenglinsky, 2002).

Knowledge. Teacher effectiveness remains one of the key factors in improving student achievement (Coleman et al., 1966, Entwisle & Alexander, 1992; Hanushek & Rivkin, 2010). Effective teachers have strong content knowledge and have insight into appropriate instructional strategies for learning (Depaepe et al., 2013; Egodawatte, 2009). Paramount in improving student achievement in mathematics is domain specific instructional knowledge (Baumert et al., 2010; Campbell et al., 2014). To explore the potential effects of teachers' mathematical knowledge on students' mathematics achievement, Hill, Rowan, and Ball (2005) conducted a longitudinal mixed-methods study involving two cohorts of students, first graders (n = 1190) and third graders (n = 1773), and 699 teachers. The sample was purposefully selected to include students and teachers from schools engaged in instructional reform. In addition, efforts were taken to obtain a sample representing high poverty, urban settings. The authors concluded that a positive relationship between student learning gains in mathematics and teacher mathematical knowledge did exist, however they were unable to decipher if the gains were truly a result of content knowledge (CK) or pedagogical content knowledge (PCK) (Hill et al., 2005).

Seeking to comparing the effects of mathematical content knowledge and pedagogical content knowledge in high school students, Baumert et al. (2010) completed a longitudinal, cross-sectional study of 181 ninth grade mathematics teachers and their students. The focus of this study was the relationship between teacher CK and teacher PCK relative to the mathematics track they teach in Germany (Baumert et al., 2010). In addition, it was hypothesized that teacher PCK would be correlated to learning gains in mathematics, particularly for lower achieving students. The PISA was administered to participating students at the end of their ninth and tenth grade years. Mathematics achievement, demographic data and teacher CK and PCK knowledge were measured. Teachers participating in the study were assessed on their mathematics CK and PCK. Open ended questions were designed to measure teachers' recognition of solution paths, awareness of student misconceptions, and instructional ability. Teachers in the study were presented with classroom scenarios and asked to provide an explanation of how they would support student learning in each situation. A mediation model was used by Baumert et al. (2010) to measure the correlation between teacher CK, PCK and student achievement. Unlike prior research, Baumert et al. (2010) conclude that teachers' PCK is more predictive of student mathematical performance than content knowledge alone. Although mathematics content knowledge is necessary, mathematics PCK is more important when learning difficulties occur (Baumert et al., 2010). The premise that PCK is more important than CK is questionable considering the results of a detailed study by Krauss et al. (2008) on discriminating between these two types of knowledge. Krauss et al. (2008) recognized a need for developing a reliable instrument to measure mathematical CK and PCK and conclude that CK and PCK are distinguishable and highly correlated ($r = .79$).

Similar studies in the United States reveal a strong association between teacher CK and PCK and mathematics achievement (Campbell et al., 2014). In a study by Jong et al. (2010) comparing reform minded practices to student achievement in grades one through six, a significant correlation ($r=.58$, $p<.01$) was reported between teacher CK and average scores on mathematics post assessments. Campbell et al. (2014) studied both the CK and PCK of 259 upper elementary and 184 middle school teachers was compared to the state mathematics achievement scores of their 6413 and 10,890 respective students. While a statistically significant

($p < .05$) relationship between teacher CK and PCK exists at both the upper elementary and middle school levels, the relationship was found to be stronger at the middle school level (Campbell et al., 2014). Because algebra is often introduced at the middle school or secondary school levels, teacher knowledge remains an important factor for improving student algebra achievement.

Pedagogical methods. Mathematical standards can outline the rigor, reform, and curriculum necessary for high school algebra students, however it is the classroom practices of the algebra teacher that have the greatest effect on student learning (Wenglinsky, 2002). In fact, Cuban (2013) suggests that when policy makers speak of improvement in educational practices, they are referring to a need for increased rigor and student-centered learning. A need for research connecting such classroom practices to student achievement was recognized by Wenglinsky (2002), as much of the research to that point only studied the effects of variables such as teacher educational attainment on student achievement. In a cross-sectional study of the 1996 National Assessment of Educational Progress data, Wenglinsky (2002) compared the mathematics scores of 7,146 eighth grade students to the classroom practices, professional development activities and education level of their teachers. Multilevel structural equation modeling was used to analyze the relationships between multiple variables, an analysis lacking in prior research (2002). Wenglinsky's (2002) research presents promising evidence that various aspects of teacher practices are associated with student achievement, even when controlling for SES. Drawbacks to this study include the inability to generalize the results outside of the eighth-grade level, yet additional research studies would soon provide similar results at the high school level.

The previously mentioned longitudinal study by Boaler and Staples (2008) is one of these studies. In addition to providing convincing evidence of the benefits of heterogeneous composition and college-preparatory curriculum for all students, this study reports that students in classroom where reform-minded practices were implemented significantly (p<.001) outperformed students in traditional, teacher centered classes on end of course exams in algebra and geometry (Boaler & Staples, 2008). Data show that students in reform oriented classrooms when from significantly behind (p<.001) on standardized assessments to significantly ahead (p<.001) compared to students in traditional classrooms. The success of students with mixed abilities in heterogeneous algebra classrooms is attributed to reformed teaching practices and a rigorous curriculum that promote achievement for all students (Boaler & Staples, 2008). The study by Boaler and Staples (2008) was extended by Horn (2008) who took a qualitative approach to studying how mathematics students responded to pedagogical methods. For students whom she calls turnaround students (students underprepared for college preparatory mathematics), Horn (2008) attributes their change in motivation to the positive, supportive environment created by the teachers. These findings corroborate the positive effects that teacher enthusiasm, concern, and care have on student achievement (Darling-Hammond, 2000).

Although the studies by Boaler and Staples (2008) and Horn (2008) support reformed teaching practices for all students, Sayeski and Paulsen (2010) caution that students with mathematics learning disabilities need explicit instruction in foundational skills to support the higher order thinking and rigor indicative of reform minded classrooms. Witzel, Mercer, and Miller (2003) also support explicit instruction for assisting student with learning difficulties as they develop the underlying skills associated with abstract algebraic concepts. Witzel et al. (2013) compared the equation solving capabilities of students taught with traditional algebra instruction against students taught with a concrete-to-representational-to-abstract (CRA) model. All sixth and seventh grade students in the study experienced learning difficulties in mathematics, yet the group taught with the CRA model significantly (p<.01) outperformed their peers in solving complex equations (Witzel et al., 2013). The findings of Witzel et al. (2013) corroborate with those of Boaler and Staples (2008), Horn (2008), and Wenglinsky (2002) who maintain that reform-minded instruction positively impacts student learning in mathematics, specifically for students with learning difficulties.

Mathematics instruction recommendations, for students with learning disabilities, support the use of reform-minded instruction in developing conceptual knowledge (Woodward & Montague, 2002) and problem-solving skills (Bottge, Rueda, LaRoque, Serlin, & Kwon, 2007). In a mixed-methods study involving 100 middle and high school students, Bottge et al., (2007) found that Enhanced Anchored Instruction, a problem-based learning strategy, impacted the problem-solving skills of students identified with mathematics learning disabilities. Researchers failed to find evidence that Enhanced Anchored Instruction affect computations skills which aligns with the findings of Sayeski and Paulsen (2010) and Witzel et al. (2003) that recommend explicit instruction for developing skills such as computation (Bottge et al., 2007).

The recommendations by the National Mathematics Advisory Panel (2008) called for further restructuring of mathematics education in the United States. The recommendations include developing classrooms of inquiry, communities of learners, and environments where teachers serve as facilitators. Such learning environments are supported by a sociocultural perspective introduced by Vygotsky (1978). The National Mathematics Advisory Panel, along with the development of the CCSSM and NCTM standards spurred educators to infuse reformed teaching practices in their mathematics instruction (CCSSI, 2010; Jong et al., 2010; NCTM, 2014).

Reformed teaching practices, those that are standards based, inquiry oriented, and student centered (Sawada et al., 2002) have been shown to have a positive association with mathematics achievement at the elementary level (Jong et al., 2010), high school level (McCaffrey et al., 2001), and college level (Sawada et al., 2002; Smith et al., 2015). Jong et al. (2010) compared the Reform Teaching Observation Protocol (RTOP) scores of 22 beginning teachers with the district-developed mathematics post test scores of their first through sixth grade students. A significant correlation ($r = .56$, $p = .05$) between these factors indicates that learning outcomes are connected to reform minded practices. This sample of beginning teachers received preservice training at institutions that also used reform minded practices, illustrating the importance of teacher training reflecting these same practices.

Similar conclusions were obtained when comparing student achievement between reform-minded and traditionally instructed classrooms at the high school level (McCaffrey et al., 2001). Researchers compared student achievement data of 5,426 tenth-grade students with the self-reported instructional practices of the students' respective 182 teachers. Achievement data were collected from open-ended and multiple-choice items on the Stanford 9 mathematics test and instructional practices were measured using the Horizon Research, Inc. teacher questionnaire (Horizon Research, Inc., 1996), designed to measure reform-minded practices (McCaffrey et al., 2001). Students were enrolled in either an integrated mathematics course, that used an inquiry approach and reform-minded curricular materials, or a traditional algebra or geometry course. Students were permitted to choose either pathway. An analysis of regression model coefficients indicated a significant ($p < .05$) relationship between student achievement on open-ended items

(.174) and multiple-choice items (.184) and teacher score on the reform teaching questionnaire (1996). An analysis of achievement scores based on background values also revealed that for students in traditional courses, poverty was predictive of achievement ($p < .01$). However, no significant relationship was found for students enrolled in integrated courses. Authors (McCaffrey et al., 2001) suggest that reform-minded pedagogy may help to reduce achievement gaps between economically diverse populations.

Expectations. An underlying premise of EST is interconnectedness of factors within and between varying systems (Bronfenbrenner, 1994). As is such, the pedagogical methods that teachers employ result from the expectations they have for their students. For students who struggle in learning algebra, and for students with disabilities, teacher expectations may reflect the belief that not all students are able to learn algebra (Horn, 2008). Investigators from the STML study (Boaler & Staples, 2008) took note of a group of students who improved their participation and engagement in their first college-preparatory mathematics classes. Horn (2008) intended to investigate the attrition of students in a college preparatory pathway and identified seven students to track throughout their high school mathematics courses. Qualitative data from classroom observations, student interviews, and teacher interviews over the course of the longitudinal study revealed that teacher expectations dictated, in part, students' continued enrollment in college preparatory mathematics classes (Horn, 2008). The rich qualitative data obtained by Horn (2008) through classroom video observations and student and teacher interviews provides evidence that the equitable, reform minded, supportive environments offered at Railside school, were responsible for the sustained enrollment of students in college preparatory classes. Students reported having more "fun" (Horn, 2008, p. 225) in rigorous college preparatory courses and expressed feelings of "hate" (p. 226) for prior remedial classes.

Students appeared to value the support of their reform-minded teachers and described these teachers as caring, helpful, and capable of making them feel “smart” (p. 226). Supportive environments were also responsible for the mathematics success of students with disabilities in settings where teachers view inclusion as a matter of social justice (Lalvani, 2013).

Students who struggle to learn algebra often encounter mesosystem level factors which continue to prevent them from achieving. A multitude of observational studies depict the effects of teacher expectations and pedagogical methods on student mathematics learning. Studies conducted on the learning tasks that elementary (Houssart, 2002) and middle school (Stein, Grover, & Hennigsen, 1996) teachers implement, report that teachers resort to providing simplified tasks that focus on product over process when students find tasks difficult. Both Houssart (2002) and Stein et al. (1996) analyzed observational records of classroom interactions and obtained similar findings. Stein et al. (1996) report a clear decline in the cognitive level of mathematical tasks throughout implementation. Houssart (2002) also reports an unintended outcome of when cognitive demands of mathematical tasks diminish. When offered non-challenging mathematics tasks, students exhibited challenging behaviors such as refusal to do work and inappropriate use of student materials. The rigorous and challenging expectations set forth by the CCSSM and NCTM standards are often reduced to foundational skills when teachers encounter students who exhibit difficulties in algebra (Powell, Fuchs, & Fuchs, 2013). Reducing cognitive demands of mathematical tasks contradicts the recommendations of researchers who support maintaining rigor for all students (Maccini & Gagnon, 2000: Powell et al., 2013).

Microsystem

At the center of the concentric environments, interacting on the ability to learn algebra, lies the student (Neal & Neal, 2013). From national and state legislature, to district-wide initiatives, and into the classroom, each decision made by those in the nested layers affects the student. The complexity of this system is deepened by microsystem level factors surrounding the student even before they enter algebra class.

Self-efficacy. An individual student's belief and judgement of their capability to perform mathematic tasks forms their mathematics self-efficacy (Bandura, 1986). More than just a personal belief, students' mathematics self-efficacy affects learning outcomes and their mathematics achievement (Hackett & Betz, 1989; House & Telese, 2008; Williams & Williams, 2010). Both student and instructional factors, as they lead to achievement in algebra, were the focus of a cross-sectional quantitative study by House and Telese (2008). The researchers analyzed data from the 2003 Trends in International Mathematics and Science Study (TIMSS). Separate analyses were conducted on TIMSS data for 4,244 thirteen-year-old students from Japan and 7,862 thirteen-year-old students from the United States, which included student self reported Likert scale responses to their mathematics beliefs and their perception of their teachers' pedagogical methods. Regression analysis yields convincing evidence that students' mathematics self-efficacy is related to their mathematics achievement (House & Telese, 2008). As the authors suggest, this study is limited to the mathematics beliefs of 13-year-old students and such a relationship would need to be studied at the high school level. Although this research reports the existence of a relationship, research indicating how mathematics self-efficacy is formed would be helpful.

A link between student self-efficacy and mathematics achievement has been established at the high school level as well. A cross-sectional quantitative study by Williams and Williams (2010) analyzed data from the 2003 cycle of the PISA and explored the reciprocal relationship between mathematics self-efficacy and mathematics achievement. The 2003 PISA included a mathematics assessment along with demographic items and was administered to 15-year-olds in 41 nations. In the United States, 5,342 students across 262 schools were assessed. Adding to the research on mathematics self-efficacy and mathematics achievement, Williams and Williams (2010) report that reciprocal determinism between these two variables is empirically supported in 26 nations, including the United States. The complexity of the variables contributing to this relationship is noted by the authors, yet such research begins to explain the powerful effects of self-efficacy on learning.

Language skills. Current research supports the conclusion that language ability is critical to mathematical development (Donlan, 2007; Morin & Franks, 2012; Nys et al., 2013). Early numeracy skills, such as counting, number naming, and basic computation, involve the language skills of phonological processing and verbal working memory (Nys et al., 2013). In a longitudinal study of kindergarteners, a significant relationship between language and early numeracy skills was reported (Toll & Van Luit, 2014). Although these conclusions are limited to young students, research also indicates that weak numeracy skills can affect students' future mathematical development (Toll & Van Luit, 2014). The research of Donlan et al. (2007) corroborates and furthermore specifies that language deficits are associated with both counting and calculation ability. In older students, mathematical problem-solving is reliant on verbal strategies, and SLI can thus affect mathematic problem-solving skills (Barnes & Raghubar, 2014).

The nature of the relationship between language and numeracy, specific to calculation abilities can be explained by how the brain processes mental calculations. Mental calculations are processed phonologically (Nys et al., 2013; Noel, Desert, Aubrun, & Seron, 2001) and stored in the verbal storage space of working memory (Noel et al., 2001). Therefore, throughout their encounters with mathematics, difficulties persist for students who have phonological processing deficits (Nys et al., 2013). A cross-sectional study by Wei, Yuan, Chen, and Zhou (2012) is one of the few studies that examines the effect of language ability on mathematics achievement in older students. Researchers collected quantitative data on 80 undergraduate students who were not mathematics majors. The study took place in Beijing and revealed that language processing affects the mathematics performance of students in advanced mathematics classes. Research studying the effects of language ability on learning algebra at the high school level is necessary.

Executive function. Executive function skills involve constructs such as attentiveness, inhibition, the control of long-term and working memory, phonological processing, and visual-spatial skills (Baddeley, 1998). Research indicates that the various constructs which comprise executive function continue to grow in number, but that dual-task studies can provide evidence on how specific executive functions relate to mathematics performance (Raghubar, Barnes, & Hecht, 2010).

Numerous studies reveal the complex nature of the association between executive function and mathematics performance (Bull & Scerif, 2001; Mazzocco & Kover, 2007; Powell et al., 2013; Swanson & Beebe-Frankenberger, 2004). The role that executive functions have in the development of mathematics skills, was the focus of a cross-sectional study by Bull and Scerif (2001). The researchers obtained demographic data, mathematics ability scores, reading ability scores, and general intelligence scores on 93 students in Primary 3 grades in Scotland. Mathematics ability was measured by the Group Mathematics Test (Young, 1970), reading ability by the British Ability Scales (Elliott, Murray, & Pearson, 1979), and intelligence by subtests of the Wechsler Intelligence Scale for Children (Wechsler, 1974). The students were individually assessed on four measures of executive functioning: Wisconsin Card Sorting Test, Stroop Test, Dual-Task performance and counting span. Correlation analysis revealed an association between mathematics achievement and specific aspects of executive function. These aspects include working memory capacity, the ability to inhibit or ignore unrelated information, and perseverance levels, the ability to sustain effort throughout a task (Bull & Scerif, 2001). Conclusions regarding the difficulty that children with low mathematics ability have in inhibiting mathematics information and learned strategies, along with maintain information in working memory, are limited to this age group. Conducting similar research with middle school and high school students could help in determining how executive functions affect students in algebra, where demands on these functions increase.

Research also indicates that not only is executive function associated with current mathematics performance, and performance over time (Mazzocco & Kover, 2007). In a longitudinal study measuring the results of the Test of Early Mathematics Ability, the Woodcock-Johnson Battery, and the Contingency Naming Test, Mazzocco and Kover (2007) explored the current and longitudinal relationship between executive function and mathematics performance. Extending the body of research in this area, the researchers were interested in determining the predictive nature of executive function skills on future mathematics ability (2007). Assessments were administered to a sample of 178 students on three occasions, during their first, third, and fifth grade years of school. Relative to future mathematics performance, a students' level of executive function skills places them at an advantage or disadvantage in learning abstract mathematics, such as algebra, a subject that is dependent on the strength of these skills (Mazzocco & Kover, 2007).

Socioeconomic status. SES has been proven to be strongly correlated to mathematics achievement (Entwisle & Alexander, 1992; Darling-Hammond, 2000) across all age groups

(Finn et al., 2016; Sheldon & Epstein, 2005; Williams & Williams, 2010). The factors of SES, working memory, and academic achievement were the focus of a cross-sectional study of 67 urban and suburban middle school students (Finn et al., 2016). Researchers obtained SES data and scores on the Massachusetts Comprehensive Assessment System from the Massachusetts Department of Elementary and Secondary Education. Students were also assessed on the Test of Nonverbal Ability. fMRI imaging was also obtained as students completed *n*-back tests and Countback tests to measure the demands of working memory. Supporting prior research, results indicate that students from higher SES performed better on mathematics achievement tests (Finn et al., 2016). The study concluded that higher SES is correlated with greater working memory capacity and an increased response to demands placed on working memory (Finn et al., 2016). This study extends what is widely known regarding the effects of SES on achievement.

Summary of Empirical Findings

The empirical research on factors contributing to low achievement in algebra offers a wide and diverse set explanations for why students struggle to succeed in this discipline. EST provides a strong framework for understanding the many factors that influence a student's understanding of algebra. Research posits that student achievement in mathematics is related to socioeconomic status (Finn et al., 2016; Williams & Williams, 2010), the presence of a language disability (Donlan et al., 2007), and decreased working memory function (Toll & van Luit, 2014), all microsystem level factors. Findings also support that microsystem level factors such early numeracy skills (Watts, Duncan, Siegler, & Davis-Kean, 2014) and computational skills (Siegler et al., 2012) predict mathematics achievement in high school.

Mesosystem level factors that affect the learning and performance of students in high school mathematics classrooms include educator content knowledge and educator pedagogical knowledge (Allensworth et al., 2009; Baumert et al., 2010; Gamoran et al., 1997). Because teacher effectiveness is a key factor in student achievement (Entwisle & Alexander, 1992), students who experience difficulties in mathematics are best served by mathematics educators who implement effective pedagogical methods (Allen et al., 2013), maintain high expectations and rigor (Burris et al., 2008), and employ student-centered learning strategies (Wenglinsky, 2002). At the exosystem level, curriculum choices (Burris et al., 2008) and educational policies impact equitable access to the mathematics curriculum (Pugach & Warger, 2001). EST can help in understanding that "Students fail(ed) mathematics for reasons other than the inability to engage with the material" (Horn, 2008, p. 221).

Responsibility lies with mathematics teachers of the 21st century to produce students who are college and career ready (National Mathematics Advisory Panel, 2008; Rakes, Valentine, McGatha, & Ronau, 2010). Current standards require that students possess the ability to reason, problem-solve, communicate, and collaborate (Eddy et al., 2015; NCTM, 2000; NCTM, 2014; National Mathematics Advisory Panel, 2008) and demonstrate a command of algebraic concepts (Kortering, de Bettencourt, & Braziel, 2005). Guidelines for effective mathematics instruction include transforming traditional teacher-centered classrooms into mathematical communities of learners where student-centered tasks direct the instruction (NCTM, 2014; Rakes et al., 2010).

Conceptual Framework

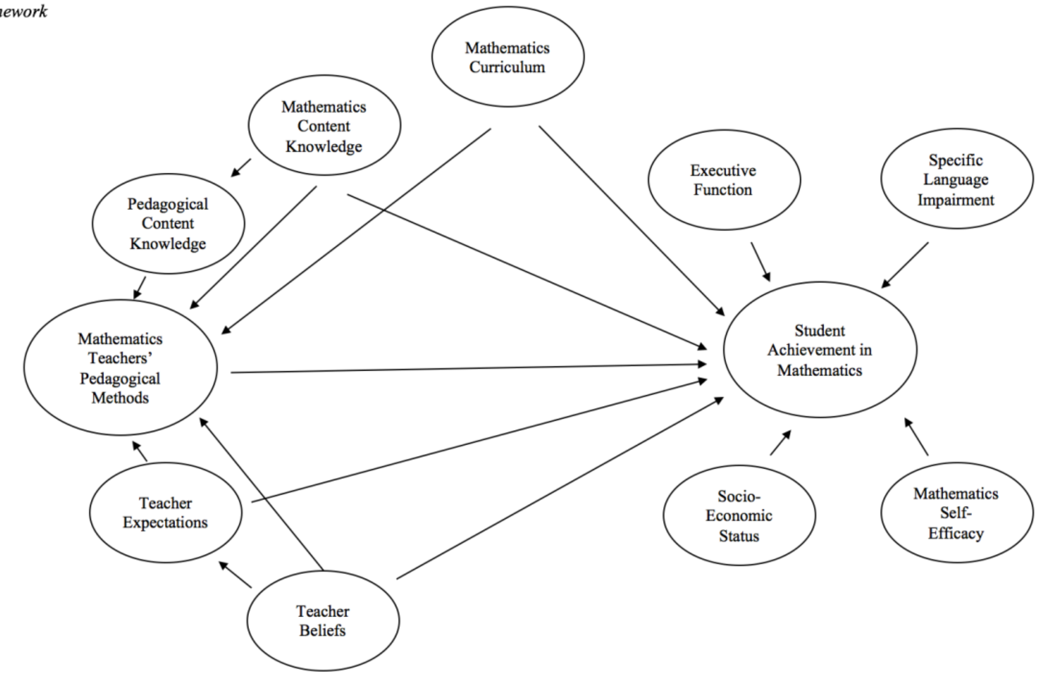

Figure 1. 2. The conceptual framework illustrating factors that impact mathematics teachers' pedagogical decisions.

Figure 1.2 provides a visual description of the conceptual framework guiding the work of this dissertation. The conceptual framework indicates the strong influence that pedagogy has on student achievement outcomes (Coleman et al., 1966, Entwisle & Alexander, 1992; Hanushek & Rivkin, 2010). Mesosystem and microsystem factors from the theoretical framework (see Figure 1.1) underscore the relationship between mathematics pedagogy and student outcomes. Pedagogical methods, moderated by teacher content knowledge, teacher pedagogical knowledge, teacher beliefs, and teacher expectations, impacts student learning which is also affected by microsystem factors.

Factors Affecting Professional Context

The effects of the factors outlined in this literature review are visible in the decline of proficiency rates for the school district examined in this study. District proficiency rates in algebra are at a historic low of 44.6% for high school students (Report Card Resources, 2017).

The high school pertinent to this study reports a proficiency rate of 26% in algebra (Report Card Resources, 2017). A greater concern exists for students with disabilities. Providing additional insight into the overall algebra achievement data, are proficiency rates for students with disabilities. At high school A, only 8.4% of SWD passed the algebra end of course exam, compared with high school B whose passage rates for SWD are 10%. When compared with scores of students without disabilities, the 31.2% and 44.4% scores for high school A and B respectively illustrate a tremendous gap in achievement for this subgroup.

Chapter 2: Needs Assessment

The literature review, organized by an EST and a conceptual framework, revealed numerous factors which affect the algebra achievement of high school students. Historically low levels of achievement in algebra are a concern for the students, staff, and parents of this suburban school district. Numerical data alone, presents a convincing argument for the magnitude of this problem. The lack of reform-minded practices in algebra classrooms further supports the problem of low algebra achievement in these high schools.

Context of Study

The professional context of this research study is a Midwestern K-12 suburban public school district with an enrollment of more than 7,750 students. Data for this needs assessment come from the two high schools in the district representing 2,484 students. The district is composed of students who are white, non-Hispanic (82.4%), Black, Non-Hispanic (8.7%), multiracial (4.9%), Asian or Pacific Islander (2.3%), Hispanic (1.5%), and American Indian (0.2%). Demographic data report that 15.9% of the students in this district are identified with specific learning disabilities and that 37.6% are on a free or reduced lunch plan.

Target Population

The stakeholders concerned with the algebra achievement of students in this district include district administrators, district educators, parents, and students. Algebra end of course exams contribute to the report card issued by the state (Report Card Resources, 2017). Student performance on end of course exams comprises 50% of the overall score on teacher evaluations (Teacher Evaluations, 2017). Individual scores on algebra end of course exams contribute to a numerical rating which enables or prevents students in this state from receiving a high school diploma (Graduation Requirement 2018 and Beyond, 2017). The most compelling argument for improving the algebra achievement levels in this district is to reduce mathematics anxiety (Meece, Wigfield, & Eccles, 1990), improve mathematics self-efficacy (Williams & Williams, 2010), and increase opportunities (Eddy et al., 2015) for the algebra students in this district.

Purpose of the Needs Assessment Study

The purpose of this needs assessment was to evaluate how mathematics teachers' instructional practices contribute to low levels of achievement in algebra, as measured by state end-of-course exams, for students in this district. The needs assessment study sought to uncover how teacher practices and pedagogical methods influence algebra achievement in the two high schools of this professional context.

Teacher expectations and pedagogical methods are strongly associated with mathematics achievement for high school students (Boaler & Staples, 2008; Horn, 2008; Wenglinsky, 2002). This assessment endeavored to gauge the self-reported understanding and implementation of reform-minded teaching practices of algebra teachers. Reform-minded practices include those that encourage students to work collaboratively and place a concentration on conceptualization, reasoning, probing, and extending knowledge (Boaler & Staples, 2008). The needs assessment also probed into teachers' awareness and desire to implement this state's Learning Standards for Mathematics and Standards for Mathematical Practices (state Department of Education, 2010).

Research Questions

To provide a better understanding of reasons why students in this district perform so poorly on algebra achievement exams, three research questions were proposed.

Research Question 1: What are the instructional beliefs of the algebra teachers in this district?

Research Question 2: What pedagogical methods are employed by the algebra teachers in this district?

Research Question 3: How are reform-minded practices implemented by the algebra teachers in this district?

Research Question 4: What association exists between the self-reported beliefs and observed practices of reform-minded pedagogical methods for the algebra teachers in this district?

Methodology

Two methods of data collection were used to inform this study. The sample was purposefully selected to include all algebra teachers from the two high schools in this district. All algebra teachers gave signed consent to participate, therefore the sample is inclusive of such. All participants completed a teacher survey, while a subgroup of these same participants participated in classroom observations. Each group of participants is described in the following section.

Participants

Teacher respondents (*N* = 8) comprised the total number of algebra teachers across the two high schools in the district. Algebra teachers who teach self-contained classes and algebra teachers at the middle school level were not invited to participate. The sample consisted of three female teachers and five male teachers. Both mathematics content teachers (*n* = 6) and interventional specialists (*n* = 2) made up this sample. Most of the teachers (75.0%) have more than 10 years of teaching experience. Of this group, all report having more than 10 years of experience in this district. The remaining teachers (25.0%) report having 2-5 years of overall teaching experience, exclusively with this district.

The experience that this sample of teachers has in teaching algebra is varied. Half of the respondents reported having 2-5 years of experience teaching algebra, one teacher reported 6-10 years of experience, and three teachers report more than 10 years of experience in teaching algebra. Most the teachers in this sample (75.0%) have obtained their master's degree, even though obtaining a master's degree is not a requirement in this state. Of those with a master's degree, three teachers reported more than 30 hours of additional graduate coursework, one teacher reports 45 hours of graduate credit beyond a master's degree, and one teacher holds a doctoral degree.

Data were collected on six participants who comprise a sub-group of those described above. These participants represent the mathematics content teachers (*n* = 6). Removing the two intervention specialists from this sample did not drastically change the overall demographics of the remaining sample. Because one intervention specialist was female and reported more than 10 years of experience teaching algebra, and one intervention specialist was male, reporting between 2-5 years of experience teaching algebra, the subgroup of participants involved in the classroom observation remains demographically similar to the total sample of participants. Two of the observations took place in co-taught classrooms and data for these observations were reported for the mathematics content teacher, who was responsible for the majority of the instruction.

Measures

The main constructs examined in this needs assessment are the implementation of reform minded pedagogical practices and teacher expectations of algebra students. Reform-minded practices include those that encourage students to work collaboratively and those that place a concentration on conceptualization, reasoning, probing, and extending knowledge (Boaler & Staples, 2008). Reform-minded practices also include those which infuse the eight Mathematics Teaching Practices outlined by the NCTM *Principles to Actions* (NCTM, 2014).

Teacher survey. A teacher survey was developed by the researcher (adapted from other surveys) and includes 39 Likert-scale items measuring the variables of teacher experience, teacher beliefs, knowledge of mathematics standards, and use of reform minded pedagogical methods. Four items measured teacher experience (1 = *zero to one years of experience* to 4 = *more than 10 years of experience*), seven items measured teacher beliefs (1 = *strongly disagree* to 4 = *strongly agree*), six items measured knowledge of this state's Learning Standards for Mathematics (1 = *strongly disagree* to 4 = *strongly agree*), and nine items measured teacher assessment practices (1 = *strongly disagree* to 4 = *strongly agree*). The remaining 12 items measured the self-reported frequency of use of reform-minded instructional practices (1 = *less than once a week* to 4 = *daily).* The full questionnaire can be found in Appendix A of this report.

Items which reflect reform-minded practices were adapted from the 2013 Teaching and Learning International Survey (OECD, 2013) and were constructed to reflect the mathematics teaching practices of *Principles to Actions* (NCTM, 2014). The questionnaire was piloted by three teachers who were not participants in this study. Cognitive interviews revealed grammatical errors, which were corrected, but no further adaptions to the questionnaire were deemed necessary. The Cronbach's alpha for the 12 reform-minded practice items was .678 which indicates a medium level of reliability (Carmines & Zeller, 1979) of this component of the questionnaire to measure reform-minded practices. The Cronbach's alpha would be raised to .704 if the item "students reflect on their learning" was removed. Upon further inspection, this item could be answered a variety of different ways depending on the teacher's interpretation of the word reflect. However, inclusion of the item had a relatively small effect on the reliability, compared to an acceptable value of .80, and retaining the item maintained alignment between the questionnaire and markers of reform-minded teaching.

Classroom observation. The self-reported frequency of implementation of reform-minded pedagogical methods in the classroom was measured with an adapted version of the Reform Teaching Observation Protocol (RTOP) (Sawada et al., 2002). This observation protocol is composed of 25 items which measure lesson design and implementation, propositional knowledge, procedural knowledge, classroom culture, and student/teacher relationships. Cronbach's alpha for each of the subscales of the RTOP range from .80 to .93 indicated the strong validity of the protocol (Sawada et al., 2002). In addition, the RTOP has been found to be predictive of the mathematical conceptual understanding and number sense of teachers who attend RTOP workshops (Sawada et al., 2002). Each item on the protocol is scored from 0 = *never occurred* to 4 = *very descriptive* for a total possible rating of 100 points. The protocol can be found in Appendix B of this report.

Data Collection

The manner in which the data were collected from the teacher questionnaires and observation protocol is described in the following section. The needs assessment employed a mixed-methods approach to analyze data collected from teacher questionnaires and classroom observations.

Teacher questionnaires. Paper copies of the teacher questionnaire were delivered to the participants' school mailboxes in sealed envelopes after written consent was obtained. Teachers were directed to return the questionnaires in the sealed envelopes provided and addressed by the researcher to the school mailbox of the researcher. The sealed envelope had no markings which revealed the identities of the respondents. To maintain anonymity, participants were assigned a numeric code (pseudonym) known only to the researcher. The numbers were assigned using a random integer generator. Four of the questionnaires were returned as directed. Follow-up emails were written to request the return of the questionnaires from the remaining four teachers. After the emails, the four remaining questionnaires were returned for a 100 percent response.

Classroom observations. Classroom observations were completed over the course of one week in May 2017. Teachers were asked to provide a preferred date for the observation and in all cases accommodations were made to meet the teacher's requested date and class period. Each algebra teacher was observed for one full period which consisted of a 45-minute time frame. The RTOP was used to document the observations and field notes were recorded to provide evidence that supported numeric scoring. Teacher comments from informal discussions, which occurred for approximately five minutes following the observation, were also recorded. Each teacher received a numeric score on the RTOP, calculated by the researcher. Code numbers were again assigned to each protocol to ensure teacher anonymity.

Findings

Results from the teacher questionnaire and classroom observations reveal varying expectations and levels of reform minded practices. Descriptive statistics were calculated using SPSS software to answer research questions one through three. Pearson calculations, also determined using SPSS software, were used to determine the degree to which teacher's self-reported practices correlate with observed practices. These results will be presented pertinent to the research questions guiding this needs assessment.

Research question 1. What are the instructional beliefs of the algebra teachers in this district? All teachers (100%) agree or strongly agree on several components of the items in the area of teacher beliefs. These include the view that teachers are facilitators of learning, that students should explore solutions before being provided solutions, and that thinking and reasoning are imperative processes in algebra. A variation ($M = 3$, $SD = 1.309$, $N = 8$) in constructing lessons where students are grouped by ability was noted. Although 25.0% ($N = 8$) of the teachers responded that they strongly disagree, 75.0% ($N = 8$) of the respondents agreed or strongly agreed that students learn best when they are in classrooms of similar ability peers. The majority of teachers indicated that they were knowledgeable of the state Learning Standards for algebra (87.5%) and stated that they consult these standards to plan their lessons (87.5%). In terms of future professional development, 50.0% ($N = 8$) of the teachers indicated that they would like further professional development on the state Learning Standards, 63.5% ($N = 8$) desire to have more training on the use of the state Standards of Mathematical Practice.

Research question 2. What pedagogical methods are employed by the algebra teachers in this district? Descriptive analysis of teachers' self-reported use of reform-minded instructional practices yielded variation. Most teachers (87.5%, $N = 8$) reported using whole class lecture three or more times per week. Teachers reported that they work through examples as a means of instruction either three to four times per week (37.5%, N = 8) or daily (62.5%, N = 8). Most teachers stated that they do not have students work on projects (87.5%, N = 8) or use manipulatives to discover or explain concepts (75.0%, N = 8). In terms of differentiation, findings indicate that teachers often give different work to students who find algebra difficult more than three times per week (50.0%, N = 8).

Research question 3. How are reform minded practices implemented by the algebra teachers in this district? Findings from classroom observations revealed that reform-minded practices and mathematics teaching practice standards are not consistently implemented in the classroom. The average RTOP score for the six mathematics content teachers was 32.2 (S =15.4) out of 100. Table 2.1 displays individual scores for the six participants. Scores ranged from 18 to 57 out of 100 with a median score of 28. Descriptive statistics indicated that most of the teachers (66.7%, n = 6) received a score of 33 or lower on this protocol. Scores between 31 and 45 on the RTOP indicate minimum student involvement and lecture oriented instruction (Ebert-May et al., 2011).

An analysis of the five subscales of the RTOP provided greater insight into the specific factors that contributed to the overall level of reform-mindedness of participants' mathematics lessons. The first subscale measures lesson design and implementation. This subscale resulted in the lowest scoring subscale for the algebra teachers in this sample (M = 5.7 and S = 5.0). Teachers also scored low on the subscale of student/teacher relationships (M = 4.8 and SD = 2.6). Items in the lesson design and implementation subscale include the instructor's use of activating prior knowledge, encouraging engagement and exploration, and creating student directed lessons (Sawada et al., 2000). The highest scoring subscale measured lesson content regarding teacher propositional knowledge (M = *8.8* and SD = 5.2). Items measuring propositional knowledge assess the instructor's fundamental and conceptual knowledge of mathematics.

Field notes provided qualitative evidence for the scores given on the RTOP. These notes provided qualitative support for understanding the factors associated with low algebra achievement for this district. The lowest scoring teacher (18) made comments such as "…the key to success is doing a problem over and over again until you get it right" and "…for problems like these, always put it in *y* equals form". In contrast, the highest scoring teacher (57) listened to the discourse of students as they collaborated and guided the instruction with "it appears that you are not clear on…" and used real-world examples when presenting problems such as "…if you were going to order a pizza." Field notes also indicated that most teachers (87.5%) model mathematics procedures, as students take notes, as the predominant method of instruction.

Research Question 4. What association exists between the self-reported beliefs and observed practices of reform-minded pedagogical methods for the teachers in this district. Table 2.1 provides a numeric summary of each participant's score on the teacher beliefs and self-reported subscales of the teacher questionnaire. Possible scores on these subscales range from 4 to 48 and 4 to 28 respectively. Table 2.1 also provides a numeric summary of each participant's overall RTOP score and their scores on each of the five subscales of the RTOP protocol; (a) lesson design and implementation, (b) propositional knowledge, (c) procedural knowledge, (d) classroom culture, and (e) student/teacher relationships. Pearson correlation calculations failed to reveal a significant correlation between self-reported practices and RTOP score or between self-reported practices and RTOP subscale scores. However, the data reveal discrepancies between self-reported practices and composite RTOP scores. The data suggest that although teachers express beliefs that they are facilitators in the classroom, that students should explore multiple solutions, and that reasoning is important, these practices are not being observed in the classroom.

Table 2. 1

Numeric Summary of Teacher Questionnaire and RTOP Data

	Teacher Beliefs Total Score (7-28)	Self-Reported Practices Score (12-48)	RTOP Score (0-100)	RTOP Lesson Design and Implementation Subscale (0-20)	RTOP Propositional Pedagogical Knowledge Subscale (0-20)	RTOP Procedural Pedagogical Knowledge Subscale (0-20)	RTOP Communicative Interactions Subscale (0-20)	RTOP Student Teacher Relationships Subscale (0-20)
Teacher 001	25	31	18	0*	0*	8	4	6
Teacher 002	24	29	23	2	8	4	6	3
Teacher 003	23	27	19	3	8	4	3	1
Teacher 004	22	43	33	6	9	6	7	5
Teacher 005	23	35	43	10	13	6	7	7
Teacher 006	22	33	57	13	15	10	11	8

*Scores of zero reflect that no elements of reform-minded instruction were present

Discussion

A focus on variation is an approach used to determine how systems are failing to meet the needs of all those they serve (Bryk, Gomez, Grunow, & LeMahieu, 2016). The needs of the students in the professional context are not being met. Data from statewide assessments (Report Card Resources, n.d.) reflect low levels of achievement for the algebra students in this district, as indicated by the 44.6% passing rate. RTOP scores, based on classroom observations, indicate that reform minded pedagogical methods are not being utilized by algebra teachers, which may begin to explain the current, low levels of student achievement in mathematics as measured by end-of-course exams. Although teachers self-report that they implement reform minded methods, as is evident in teacher questionnaire responses, there is a discrepancy between these self-reported scores and the RTOP scores gathered in classroom observations. Teachers did indicate a desire for further training on implementing the Mathematical Practices of the state Learning Standards, standards which align with reform minded pedagogy. Teacher PCK is associated with student achievement in mathematics (Campbell et al., 2015) and training on reform minded practices can potentially assist this district in improving algebra achievement.

Qualitative data, collected from four classroom observations at high school A began to explain a possible cause for low high school algebra achievement scores for this professional context. Of note, and the focus of this study, is a lack of reformed teaching practices. Participant observation records, from classroom observations conducted by the researcher, indicate a lack of engagement by the students in their respective algebra classes. During the four observations of two algebra teachers it was noted that the main method of instruction was lecture. Both teachers worked examples of concrete problems at the board while students copied the problems into their notes. During all four observations, students did not contribute to the development of the concept, students did not participate in classroom discourse, and the prior knowledge of students was not activated. Student interactions were limited to answering concrete, often multiple-choice items. The rigor of the algebra curriculum needs to be supported by mathematical practices that are reform-minded (CCSSM, 2010; NCTM, 2014). The participation observations showed little evidence that reform-minded practices were being implemented.

Results of this needs assessment confirm research findings that suggest that contradictions exist between teachers' self-reported and observed instructional practices (Ebert-

May et al., 2011; Huinker & Hedges, 2015; Jong et al., 2010). Although the algebra teachers in this district report using reform-minded instructional practices, mean RTOP scores (M = 32.2; S = 15.4; range = 0 – 100; n = 6) indicate that teachers generally employed lecture-oriented practices (Ebert-May et al., 2011). The algebra teachers in this district expressed a desire to learn more about the state Standards of Mathematical Practice and would benefit from professional development targeting mathematics pedagogy.

Chapter 3: Intervention Literature Review

A Sociocultural Perspective on Mathematics Instruction

A sociocultural perspective of learning places emphasis on the interactions between learners as they develop conceptual understandings (Cobb, 1994; Kozulin, 2004). Sociocultural theorists purport that a student's cognitive development is reliant on their social interactions with more knowledgeable peers and teachers (Vygotsky, 1978). In his seminal work on the development of cognition, Vygotsky (1978) stresses the need for developmentally ready students to be supported in their learning experiences to reach new levels of cognition. Furthermore, authentic learning experiences help bridge the gap between procedural and conceptual knowledge (Ginsburg, 1997).

Vygotskian theory, when applied to the mathematics classroom, posits that activity plays a central role in students connecting procedural skills with conceptual understanding (Kozulin, 2004). In fact, strategies for effective mathematics instruction align directly with the tenets of sociocultural theory. The NCTM publications of *Principles and Standards* (2000) and *Principles to Actions* (NCTM, 2014) present a rigorous set of standards that serve as guidelines for quality mathematics instruction. These guidelines encourage the use of instructional practices that promote a high level of student engagement, collaboration and communication between students, the development of reasoning and problem-solving skills, and result in a deep understanding of conceptual knowledge of mathematics (Eddy et al., 2015; NCTM, 2000; NCTM, 2014; Star et al., 2015).

Many factors within the mathematics classroom environment affect the ability of students to acquire mathematical knowledge. Teacher effectiveness, however, has been shown to mediate some of these variables and exert a strong influence on student achievement (Coleman et al., 1966, Entwisle & Alexander, 1992; Hanushek & Rivkin, 2010). Choice of pedagogical methods while upholding of rigorous expectations determines teacher effectiveness (Boaler & Staples, 2008; Wenglinsky, 2002). The needs assessment data described in chapter 2, suggest that the algebra teachers of this professional context predominantly use teacher-centered methods and focus solely on procedural knowledge. The traditional pedagogy implemented by the algebra teachers may begin to explain why low levels of achievement in algebra, as measured by state end-of-course exams, remain a problem for the district of this professional context (Clark, 2017).

A Synthesis of Literature Related to Pedagogical Interventions

Researchers agree that both procedural knowledge and conceptual knowledge are necessary for a solid understanding of mathematics (Rittle-Johnson & Schneider, 2015; Star, 2005). In their seminal work on the topic, Hiebert and LeFevre (1986) describe conceptual knowledge as a network of individual pieces of information and the relationships between them. Building conceptual knowledge involves recognizing how individual pieces contribute to the network and bridging prior and current knowledge (Hiebert & LeFevre, 1986). Conversely, procedural knowledge relies on an understanding of the symbols, the language of mathematics, and the rules and algorithms involved in manipulating the symbols solve problems (Rittle-

Johnson & Schneider, 2015). NCTM (2014) recommends that students' conceptual understanding provides the groundwork for their procedural fluency.

Neuroscience offers insight into how learning occurs in the brain and evidence from the field supports the importance of allowing students to see how pieces of information factor into the whole (Bruer, 2008). Research on mathematics instruction also purport the benefits of mathematics instruction focused on conceptual understanding (Carraher et al., 2006; Donlan et al., 2007; Nathan & Koedinger, 2000; NCTM, 2014). Students with mathematics disabilities and mathematics difficulties, who struggle with procedural accuracy (Donlan et al., 2007; Sherman, Walkington, & Howell, 2016), respond positively to instruction with a conceptual focus. The following literature review will focus on specific instructional strategies and broad pedagogical practices designed to improve students' procedural and conceptual understanding of mathematics, thereby increasing student performance in mathematics.

Intervening through Specific Instructional Strategies

Using manipulatives to enhance understanding. In a meta-analysis of research on interventions in algebra Rakes et al. (2010) identified five main categories of intervention. Studies of all the categories, including curricula, instructional change, manipulatives, technology tools, and technology curricula, reported significant ($p<.05$) effect sizes on improving student achievement in algebra (Rakes et al., 2010). Furthermore, Rakes et al. (2010) recommend the use of manipulatives and technology to improve student achievement in mathematics. Some mathematics educators express concern that instructing students using multiple representations can create confusion and difficulty in transfer, especially for students who have mathematical difficulties (Larbi & Mavis, 2016; Lynch & Star, 2013). However, introducing concrete representations is a recommended strategy for teaching mathematics to students with disabilities

(Maccini & Hughes, 2000; Strickland & Maccini, 2010). The use of multiple representations provides scaffolding for students (Vygotsky, 1978) as they move from a concrete stage of development to an unfamiliar abstract stage.

Manipulatives, concrete objects used explore mathematical relationships, are instrumental in helping students improve their conceptual understanding of mathematics (NCTM, 2000) and assist students in forming connections between conceptual and procedural knowledge (Agrawal & Morin, 2016). Larbi and Mavis (2016) conducted a quasi-experimental study to investigate how the use of algebra tiles would affect the understanding of the concepts of distributive property and factoring among junior high algebra students. The experimental group (n = 25) received instruction on factoring and distributing with monomials and binomial that was visually represented (through the use of algebra tiles) and algebraically represented (paper and pencil problems). The control group (n = 25) received the algebraic representation alone. A t-test of comparison of posttest mean scores reported a significant (p = .00) difference between the groups (Larbi & Mavis, 2016). Researchers report that the use of algebra tiles not only contributed to the ability of students to visualize the concepts of factorization and distribution of algebraic expressions, but the manipulatives engaged the students and sustained their interest in the lesson. Although improved engagement was not an intended variable in this study, cognitive science supports engagement for improved attention and learning (Farrington et al., 2012; Skinner, 1965)

Concrete manipulatives offer a tangible representation of mathematical concepts and have been found to be an important factor in teaching mathematics (Wilson, Cooney, & Stinson, 2005), but other forms of representation are equally effective. Satsangi, Bouck, Taber-Doughty, Bofferding, and Roberts (2016) found virtual manipulatives to be equally as effective as concrete manipulatives for teaching linear algebraic problem-solving. In a single-subject, alternating treatment design Satsangi et al. (2016) found that students with mathematics disabilities ($n = 3$) had greater accuracy (mean = 90%) when using concrete or virtual manipulatives then when asked to solve algebraically represented problems (mean baseline range 0% - 50%). The small sample size limits the strength of the conclusion (Satsangi et al., 2016) yet the results corroborate the effectiveness of using manipulatives to support concrete to abstract representations (Strickland & Maccini, 2013). Cognitive-based interventions, such as concrete to abstract representations were found to have some of the largest effect sizes on student achievement in mathematics for students with disabilities (Hughes, Witzel, Riccomini, Fires, and Kanyongo, 2104; Maccini & Hughes, 2000). These results could lead to the development of instructional strategies that contribute to far transfer, the transferring of knowledge regarding one concept to similar mathematical concepts (Barnett & Ceci, 2002).

For students with mathematics disabilities the use of manipulatives requires purposeful planning (Maccini & Gagnon, 2000). In two separate literature reviews regarding mathematics instruction for students with learning disabilities, Myers, Wang, Brownell, and Gagnon (2015) and Maccini and Gagnon (2000) posit that physical and virtual manipulatives support the development of conceptual knowledge from the concrete to the abstract. However, it was not the use of manipulatives alone that accounted for improved problem solving (Maccini & Gagnon, 2000). Improved student performance resulted from the use of manipulatives paired with scaffolded inquiry (Myers et al., 2015) or student explanation (Maccini & Gagnon, 2000). Furthermore, recent research supports the inclusion of manipulatives as a specified accommodation in IEPs for students with mathematics disabilities (Bouck & Park, 2018).

Analyzing worked examples. In a recent publication of strategies designed to improve algebra instruction, the United States Department of Education Institute of Education Sciences (IES) recommends instructional strategies that engage students in reasoning activities (Star et al., 2015). Compared to traditional forms of mathematics instruction where students copy worked examples from the board, IES recommends providing students the opportunity to analyze solved algebraic problems, or worked examples (Booth et al., 2015), to connect algebraic procedures to algebraic concepts as well as to identify misconceptions and common errors (Star et al., 2015). Empirical research supports the use of worked examples to supplement instruction in the algebra classroom (Booth, Lange, Koedinger, & Newton, 2013; Booth et al., 2015; Lynch & Star, 2013).

Mathematics teachers commonly use worked examples as a method of mathematics instruction (Carroll, 1992; Sweller, Chandler, Tierney, & Cooper, 1990). Extending the research on the benefits of using worked examples in mathematics instruction, Booth et al. (2013) conducted a study involving two experiments to determine how the use of correct and incorrect worked examples with students in instruction can support procedural and conceptual knowledge of solving two-step equations. In the first experiment, students were divided into three treatment groups and one control group. The first treatment group (n=30) received correct worked examples, the second treatment group (n=31) received incorrect worked examples, and the third treatment group (n=25) received both correct and incorrect worked examples. Students in the control group (n=30) received additional guided practice problems instead of worked examples. A comparison of pre- and post-test data revealed that students in the treatment group outperformed those in the control groups on procedural knowledge ($p<.07$) and conceptual knowledge ($p<.05$) (Booth et al., 2013). The second experiment within this study attempted to distinguish whether the use of correct or incorrect worked examples were more effective at improving student understanding. Results indicate that providing students with incorrect worked examples, either alone or in conjunction with correct worked examples, led to a higher conceptual understanding of solving two-step equations (Booth et al., 2013).

The effectiveness of using worked examples to increase conceptual understanding of a single algebraic standard (Booth et al., 2013) led to a year-long study focusing on the use of worked examples across a comprehensive algebra curriculum (Booth et al., 2015). The researchers studied the use of AlgebraByExample, a worked example supplement to the algebra curriculum with 380 students, in 25 classrooms, across five states. AlgebraByExample assignments contain both correct and incorrect examples, supported by the findings of Booth et al. (2013), along with written prompts designed to target commonly held misconceptions. Experimental classrooms, those receiving workbooks with the worked examples, and control classrooms, those receiving workbooks with tradition algebra problems, were matched by teacher to control for teacher variability (Booth et al., 2015). Results indicate that students who used the AlgebraByExample study assignments scored higher on tests of conceptual knowledge, procedural knowledge, and state standardized algebra assessments than their peers in the control group (Booth et al., 2015). Given that curricula materials nationwide often lack alignment to end-of-course assessment expectations (Sherman et al., 2016), the increase in state assessment scores (Booth et al., 2015) provides a rationale for using worked examples to supplement algebra instruction.

When studying the effectiveness of providing students with multiple strategies to solve algebraic problems, Lynch and Star (2013) used worked examples to assist students in making connections between key algebraic concepts. Unlike the studies by Booth et al. (2013) and Booth et al., (2015) that used a combination of correct and incorrect worked examples, the exploratory study by Lynch and Star (2013) provided students with multiple versions of correct worked examples. The worked examples served as supplementary material to the standard curriculum in 12 middle and high school algebra classes and provided students with prompts to elicit discussion on understanding, comparing, and connecting algebraic conceptions (Lynch & Star, 2013). The authors conducted semi-structured interviews with 23 students who represented a wide range of ability. Researchers presented an analysis of six of these interviews in which students identified perceived advantages and disadvantages. Students reported that initial concerns over being confused by the multiple strategies dissipated after several lessons (Lynch & Star, 2013). All students (*n*=6) reported that the multiple strategies in the worked examples provided them with different ways to solve the problems (Lynch & Star, 2013). It is not clear from this small, exploratory study whether the perceived advantages resulted from the worked examples or from the ensuing discussions (Lynch & Star, 2013). Yet together with the findings by Booth et al. (2013) and Booth et al., (2015) this study provides additional evidence for the benefits of studying worked examples.

Worked examples are a source of scaffolding in algebra instruction, reducing the demands on working memory for procedural tasks (Booth et al., 2015). However, researchers caution that using worked examples as a primary source of instruction limits the ability of students to build schemata that can be generalized to unparalleled problems (Lim & Moore, 2002). The benefits received from having students analyze worked examples in geometry, for example, corresponded to the cognitive demands of the problems (Sweller et al., 1990). Researchers compared the use of conventional worked examples and modified worked examples to processing time needed to solve coordinate geometry problems in 48 ninth grade students (Sweller et al., 1990). Compared to the conventional worked examples, the modified worked examples included geometric diagrams embedded with the work. When cognitive demands require that students attend to various components of a problem, conventional worked examples failed to further conceptual understanding (Sweller et al., 1990). Rather, modified worked examples, examples with diagrams indicating a sequence of steps required less processing time and led to increased understanding (Sweller et al., 1990).

Cognitive research suggests that the benefit of using worked examples as a source of mathematics instruction comes from the creation and resolution of cognitive dissonance (Mevarech & Kramarski, 2003). Furthermore, mathematical discourse, resulting from the accompanying analyses of worked examples leads to accurate problem-solving skills (Mevarech & Kramarski, 2003). Similarly, Lim and Moore (2002) purport that discourse between students, which accompanies analyses of worked examples in geometry, leads to accurate problem-solving skills (Lim & Moore, 2002). Sociocultural theory recognizes the importance of discourse, collaboration, and scaffolding to cognitive development (Kozulin, 2004) and supports instructional activities that provide these opportunities.

Applying metacognitive strategies. Along with developing a strong conceptual understanding of algebra, students benefit from identifying and correcting their misconceptions. Misconceptions originate at various grade levels (Egodawatte, 2009; Karp, Bush, & Dougherty, 2014; Karp, Bush, & Dougherty, 2015) and affect performance in high school algebra and advanced mathematics courses (Siegler et al., 2012; Watts et al., 2014). Metacognitive strategies, such as self-explanation, or self-talk, allow learners to construct knowledge through a manipulation of prior and new information (Chi, 2000). Methods that encourage students to verbalize their learning (external speech), ultimately leading to self-explanation (internal speech), a process supported by sociocultural theory (Vygotsky, 1978). Teachers can develop this self-regulation behavior in students, and also strengthen working memory function (Chi, 1978) by providing scaffolding through verbal prompting (McEldoon, Durkin, & Rittle-Johnson, 2013) and written prompting (Hutchinson, 1993). Empirical research supports the use of self-explanation for increasing procedural knowledge, conceptual knowledge (Hudesman et al., 2013; McEldoon et al., 2013), for improving reasoning (Maccini & Hughes, 2000) and for developing problem-solving skills (Hutchinson, 1993). Furthermore, self-explanation provides support for students across age groups and ability levels (Hudesman et al., 2013; Hutchinson, 1993; McEldoon et al., 2013)

The use of verbal and written prompts to encourage self-explanation was the focus of a mixed-methods study by McEldoon et al. (2013). Researchers (McEldoon et al., 2013) compared the effects of self-explanation prompting against solving problems without prompting for mathematical equivalence problems (e.g., $2 + 3 + 6 = \_ + 6$). Students in the treatment group ($n = 21$) were prompted to explain *how* fictitious students obtained the correct solution and *why* sample solutions were either correct or incorrect. Students in the first control group ($n = 17$) received the same number of problems to solve without prompting. Students in the second control group ($n = 31$) received additional practice problems to equate to the time necessary for self-explanation (McEldoon et al., 2013). Procedural and conceptual knowledge was compared using pre- and post-test scores on researcher developed assessments. Compared to the control group, students who were provided the opportunity to self-explain performed better on tests of conceptual knowledge (McEldoon et al., 2013). On tests of procedural knowledge, however, no differences were found between the groups. These results suggest that additional practice problems could enhance procedural skills equally as effectively as self-explanation (McEldoon et al., 2013). The authors note that students did not receive feedback on their self-explanations

(McEldoon et al., 2013). Constructive feedback is essential to effective instruction (Stein, Engle, Smith, & Hughes, 2008) and may have strengthened the results of this study.

Written prompts were also found to be effective at improving problem-solving skills in adolescents (Hutchinson, 1993). Research on junior high school students with mathematics learning disabilities confirms the benefits of self-explanation in developing problem-solving skills (Hutchinson, 1993). Twenty junior high students with mathematics learning disabilities were divided into intervention (n = 12) and control (n = 8) groups for their small group instruction. The treatment group worked on problems involving relations, proportions, and writing two-by-two systems of equations using self-question prompt cards and structured worksheets (Hutchinson, 1993). The control group worked on the same problems without the use of these aids. Data were collected from pre- and post assessments containing near transfer items (problems with similar structure) and far transfer items (problems that required generalization) (Hutchinson, 1993). Additional data were collected from think-aloud protocols, metacognitive interviews, and assessments measuring maintenance, given six weeks after instruction. An analysis of covariance revealed that students receiving the intervention scored significantly better ($p<.05$) on all assessment measures (Hutchinson, 1993). The prompt cards and structured worksheets provided students support as they developed schemata for problem-solving (Hutchinson, 2013). Using prompt cards can effectively scaffold instruction for learners and can lead to internalized speech.

Mathematics discourse creates opportunities for students to explore algebraic concepts and connect prior knowledge with new understandings. NCTM (2014) recommends that student engage in mathematical discourse. Developing their ability to do so through self-explanation (McEldoon et al., 2013) and written prompts (Hutchinson, 2013) can lead to productive discussion with peers, which is important for mathematics learners of all abilities (Stein et al., 2008).

Intervening through Pedagogical Practices

Student achievement in mathematics depends largely on teacher effectiveness (Campbell et al., 2014; Coleman et al., 1966; Hanushek & Rivkin, 2010; Wenglinsky, 2002). Positive results occur when pedagogical methods also focus on conceptual understanding (Boaler & Staples, 2008; Eddy et al., 2015; National Mathematics Advisory Panel, 2008; NCTM, 2014). Reform-minded instruction is associated with improved student achievement in mathematics (Eddy et al., 2015; Holt, Young, Keetch, Larsen, & Mollner, 2015; NCTM, 2014) and benefits learners across grade and ability levels (Sawada et al., 2002). Furthermore, reform-minded instruction involves a student-centered learning approach where students are actively engaged in constructing and sharing their knowledge of mathematics, thereby developing a conceptual understanding. Sociocultural constructivism purports that knowledge is gained when students participate in learning tasks with their peers (Cobb, 1994).

Implementing reform-minded instruction. Empirical research supports the use of reform-minded instruction and reform-minded curricula as an intervention for improving student achievement in mathematics (Krupa & Confrey, 2017; Sherman et al., 2016). In a large-scale, quasi-experimental study, researchers compared the effects of reform-minded instruction on the mathematics achievement of 19,526 high school algebra students throughout North Carolina (Krupa & Confrey, 2017). Students were enrolled in either a traditional, Algebra I or Algebra II course or an integrated mathematics course that combines algebraic, geometric, trigonometric, and statistical concepts. The integrated mathematics courses utilized the reform-based curriculum *Core-Plus Mathematics.* Educators who taught the integrated courses received training through the North Carolina Integrated Mathematics Project which sought to establish a community of learners among the teachers implementing the integrated curriculum. Researchers used hierarchical linear modeling when comparing data from state end-of-course exams in Algebra I and Algebra II. Authors concluded that students enrolled in the integrated courses using the reform minded curriculum performed better on end-of-course exams in Algebra I, even when controlling for student (SES) and school (low-performing) characteristics. Although the analysis reported no difference between the test scores of students in the treatment and control groups for Algebra II, Krupa and Confrey (2017) attribute this to a lack of alignment between the *Core-Plus* materials and the end-of-course exam. The conclusions corroborate with findings by Sherman et al. (2016) who state that a lack of alignment between recommended standards, such as the Common Core State Standards for Mathematics, and curricula continues presents problems for schools that desire reform (Sherman et al., 2016).

Reform-minded pedagogy is equally as important as reform-minded curriculum concerning student achievement in mathematics. In fact, even when presented with reform-minded curriculum materials, teacher beliefs regarding instructional practices take precedence over resources in determining the manner in which students are taught (Roehrig & Kruse, 2005; Sherman et al., 2016). Reforming mathematics education requires not only changes in standards and curriculum, but changes in pedagogy as well (Kulm & Stuessy, 1992).

Lawson, Benford, Bloom, & Carlson (2002) studied the effects of reform-minded pedagogy on student achievement in both science and mathematics. In the mathematics component of this study, research compared the computational skills, number sense, and conceptual understanding of preservice elementary teaching students to the level of reform-minded instruction implemented in their classrooms (Lawson et al., 2002). The Reformed

Teaching Observation Protocol (RTOP) (Piburn & Sawada, 2000) was used to measure the implementation of reform-minded instruction. Results indicate that student conceptual understanding of number sense strongly correlates ($r = .94$, $p < .001$ and $r = .90$, $p < .001$ respectively) with instructor RTOP score (Lawson et al., 2002). Researchers note that the lack of correlation between computational skills and RTOP score is not surprising because reform-minded instruction focuses largely on conceptual knowledge (Lawson et al., 2002).

Studies at the high school (Roehrig & Kruse, 2005) and elementary school level (Jong et al., 2010) confirm positive effects of reform-minded instruction on student learning. Jong et al. (2010) examined the relationship between the RTOP scores of 22 beginning teachers and their students' scores on district course post assessments. Teachers were all first or second year teachers who graduated from programs that also stressed reform-minded practices. Jong et al. (2010) found that reform-minded instruction correlates (r=.56, p<.05) with pupil performance in mathematics. These results corroborate with those of Holt et al. (2015) and Sawada et al (2002) who uphold that mathematics learning improves when reform-minded teaching is implemented.

Reform-minded instructional approaches promote mathematics achievement for students with mathematics disabilities (Myers et al., 2015). Effective interventions for students for with disabilities include (a) designing lessons that require student collaboration, (b) providing opportunities for self-reflection, and (c) encouraging students to use multiple problem and solution representations (Hodara, 2011), all hallmarks of reform-minded teaching. Although Gersten et al. (2009) stress the importance of interventions that provide explicit instruction for students with mathematics disabilities, they also support instruction that develops conceptual knowledge. Powell et al. (2013) also recommend interventions that develop students' conceptual understanding while scaffolding instruction based on foundational skills.

Using professional development to change pedagogy. Implementing reform-minded pedagogy requires a shift from the traditional methods to which many teachers are accustomed (Roehrig & Kruse, 2005). Also, pre-service instruction often follows traditional, teacher-centered practices and teachers tend to use instructional methods that align with how they were taught (MacIsaac & Falconer, 2002). Professional development (PD) targeted at supporting instructional improvement, rather than simply informing teachers of instructional practices, creates opportunities for teachers to transform their instructional practices (Stein, Smith, & Silver, 1999).

Research shows that PD can alter teachers' beliefs and practices and result in greater understanding and learning gains for students (Amrein-Beardsley & Popp, 2012; Borasi, Fonzi, Smith, & Rose, 1999; Borko et al., 2005). However, pedagogical changes do not occur immediately and require sustained, long-term PD to maintain fidelity of implementation (Newman, Marks, & Gamoran, 1996). A main tenet of sociocultural theory is the practice of scaffolding student learning (Vygotsky, 1978). Borasi et al. (1999) recommend scaffolding teacher learning through the creation of teacher communities to support learners as they implement changes in instructional practices.

Changes in instructional practices result from an increased awareness of effective instructional strategies. Borasi et al. (1999) examined the effects of PD on reforming the instructional practices of middle school content teachers and special educators. The PD program included a six-day summer institute experience and supported field experiences throughout the school year (Borasi et al., 1999). These components align with the guidelines for effective mathematics PD suggested by Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003). Loucks-Horsley et al. (2003) The summer institute provided 54 middle school teachers the opportunity to read about, discuss, and prepare inquiry based lessons for students across ability levels. The field experiences, completed by only 39 of the original participants, included implementing inquiry lessons, weekly meetings with support teams, attendance at full day workshops through the year, and writing reflections about the experiences (Borasi et al, 1999). Authors conclude that the PD program effectively initiated the process of change in instructional practices (Borasi et al., 1999). Descriptive statistics reveal that teachers sustained the implementation of inquiry units for up to 24 weeks in at least one of their classes (Borasi et al., 1999). Teacher reflections provide anecdotal evidence that the program increased their awareness of inquiry learning as an effective strategy for learners of all abilities (Borasi et al, 1999).

Effective PD for teachers of mathematics provides the opportunity for teachers to build pedagogical content knowledge, reflect on their learning, practice their new understanding, and collaborate with colleagues (Loucks-Horsley et al., 2003). Researchers implemented these guidelines in the creation of a pilot PD component of the Supporting the Transition from Arithmetic to Algebraic Reasoning (STAAR) program (Borko et al., 2005). In the initial phase of this mixed-methods study, 16 middle and elementary school teachers participated in a two-week summer institute focused on developing algebraic thinking among students. Components of the PD included increasing teachers' understanding of algebraic concepts, the development of pedagogical content knowledge, participation in a professional learning community, and learning in a reform-minded setting (Borko et al., 2005). Quantitative data show only a modest increase in content knowledge based on pre- and post-assessments, yet a scoring rubric calculating the array of strategies that teachers used to solve problems indicates that teachers increased their ability to use multiple strategies to solve problems (Borko et al., 2005). Self-reported qualitative data, from teacher reflections, final papers, and interviews, suggest that teachers gained new understandings, new techniques, and confidence in implementing strategies that promote algebraic thinking (Borko et al., 2005).

The studies by Borasi et al. (1999) and Borko et al. (2005) begin to provide evidence that well-designed PD positively influences teacher beliefs regarding reform-minded instructional practices and increases the likelihood that reform-minded practices will be implemented. However, researchers Adamson et al. (2003) extend such findings and report that educating teachers in reform-minded instructional practices not only increases levels of implementation, but is also associated with an increase in student achievement. Although the participants in this study included high school biology teachers and their students, Adamson et al. (2003) report an increase in students' ability to reason and understand biology concepts in classrooms where reform-minded instruction is implemented. The ability to reason and conceptualize affect a students' understanding of algebra (NCTM, 2014, National Mathematics Advisory Panel, 2008), therefore replication of this research in mathematics is warranted.

Components of Effective Professional Development

Changes in educational policies and efforts to improve education in the United States must be accompanied by high-quality teacher PD (Guskey, 1986; Tyack & Cuban, 1995; Saderholm et al., 2017). Citing a lack of conceptualization regarding characteristics of effective PD, Desimone (2009) proposed a framework for measuring the effects of PD on student outcomes and improved instruction (see Figure 3.1). Desimone's (2009) model attempts to explain the black-box (Leviton & Lipsey, 2007) of PD and outlines the features of PD that lead to changes in instruction and result in positive student outcomes. Core features of the model align with research findings suggesting that effective professional development programs create authentic opportunities to learn including (a) situating the learning in context (Learning Forward, 2011; Raphael et al., 2014), (b) providing opportunities for collaboration (Desimone, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001), (c) promoting a learner-centered approach (Darling-Hammond, Hyler, Garner, & Espinoza, 2017; Learning Forward, 2011), and (d) supporting teachers in their improvement efforts (Calvert, 2016; Darling-Hammond et al., 2017; Jensen, Sonnemann, Roberts-Hull, & Hunter, 2016).

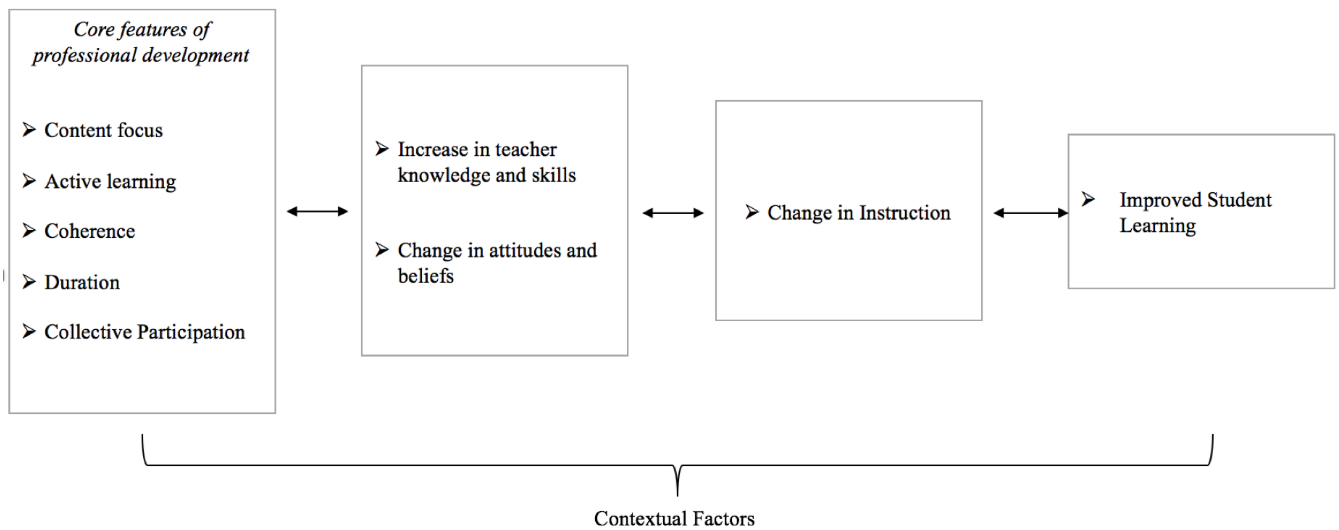

Figure 3. 1. Professional development evaluation framework (Desimone, 2009)

Applied to teacher professional development, sociocultural theory posits that learning results from shared cognition in public and private spaces (Gavelek & Raphael, 1996). Just as children use language as a bridge between developmental stages (Vygotsky, 1978), educators rely on internal and external reflection to transform personal instructional beliefs and practices (Mezirow, 1978). A more formalized version of the zone of proximal development (Vygotsky, 1978), the Vygotsky space, serves as a model describing a continuum of zones through which educators develop their expertise (Raphael et al., 2014). Moving through public, private, social, and individualized spaces, educators observe, share, practice, discern, and adapt instructional techniques (Raphael et al., 2014). Consistent with sociocultural theory, context mediates learning (Rohlwing & Spelman, 2014) supporting the recommendation to situate professional development within the school context and embed learning opportunities throughout the school day (Jensen et al., 2016; Learning Forward, 2011).

Situated

Research on effective PD promotes situating teacher PD in the professional context and among colleagues (Webster-Wright, 2009). In fact, Webster-Wright (2009) views exploring, discussing, and reflecting with colleagues as a sociocultural activity that supports adult learning as well as student learning (Vygotsky, 1978). Collective participation (Blank, de las Alas, & Smith, 2007; Garet et al., 2001), involving teachers from the same grade level or department, increases the fidelity of implementation of learned strategies. Providing collaboration, discussion, and reflection time for collective groups of teachers is a critical aspect of effective PD programs (Saderholm et al., 2017).

Situating, or embedding, professional learning in context is often the differentiating factor between effective and ineffective professional learning, especially in secondary education contexts (Fisher & Frey, 2014). Situated, site-based PD provides a necessary contextual base for teacher learning (Guskey, 2003). In an analysis of 13 studies that outlined the characteristics of effective PD, Guskey (2003) reports that the majority of the studies recommend on-site PD. Site-based PD that attends to contextual as well as educator needs provides a safe environment for teachers to inquire, experiment, collaborate, and reflect (Guskey, 2003). Guskey's (2003) findings corroborate with a recent review of the literature by Lauer, Christopher, Firpo-Triplett, and Buchting (2014). Authors (Lauer et al., 2014) examined 23 short-term, face-to-face PD programs designed to determine the characteristics of effective PD. A commonality among the studies suggests that on-site PD impacts teacher beliefs and instructional behaviors (Lauer et al., 2014).

The transformation from traditional to reform-minded mathematics instruction may require a substantial change in beliefs and practices for educators. Professional learning programs designed to facilitate this transformation require thoughtful consideration of participant and student needs (Fisher & Frey, 2014; Guskey, 2014). Current educator beliefs and practices will dictate whether an incremental or substantive change will be required (Spillane, Reiser, & Reimer, 2002). Situated professional learning promotes individual growth (Clarke & Hollingsworth, 2002) thereby impacting the instructional practices and beliefs of this sample of mathematics educators.

Learner-Centered

Just as mathematics reform efforts emphasize student-centered instruction for optimal mathematics achievement, researchers recommend learner-centered PD to transform instructional practices (Loucks-Horsley et al., 2003). The previously mentioned literature reviews by Guskey (2003) and Lauer et al. (2014) support active learning opportunities and participant engagement as essential to improving PD outcomes. Darling-Hammond et al. (2017), in their report for the Learning Policy Institute maintain that active learning experiences, an essential component of PD, contribute to changes in teachers' practices.

Researchers find that when teachers engage as learners in instructional strategies they gain confidence and are more likely to implement the strategies in their classrooms (Prather & Brissenden, 2009). In a qualitative study aimed at gaining teachers' perspectives on PD, Bayar (2014) found that teachers appreciate active engagement and the opportunity to practice learned strategies. Not only do active learning increase teachers' skills, but engagement leads to changes in instructional practices (Garet et al., 2001). Furthermore, when assessing the extent to which participation in PD results in a change of pedagogy, Singer, Lotter, Feller & Gates (2011) conclude that PD must include components that immerse, engage, and support learners to deepen pedagogical knowledge.

A more formalized version of the zone of proximal development (Vygotsky, 1978), the Vygotsky space, serves as a model describing a continuum of zones through which educators develop their expertise (Raphael et al., 2014). Movement through the Vygotsky space occurs naturally in learner-centered approaches to professional development (Clarke & Hollingsworth, 2002; Gavelek & Raphael, 1966). In these spaces, educators enact and reflect on learned content leading to instructional change (Clarke & Hollingsworth, 2002; Guskey, 2002). Schools that adopt learner-centered approaches to professional learning witness an increase in teacher agency leading to a culture of collective responsibility (Calvert, 2016; Jensen et al., 2016). A sense of collective responsibility is crucial to sociocultural theory where learners facilitate one another's growth (Vygotsky, 1978).

Collaborative

High-performing school systems recognize that effective educators are perhaps one of the district's most significant resources when it comes to advancing professional learning (Jensen et al., 2016; Learning Forward, 2011). Professional learning designs that incorporate collaboration among teachers elevate the expertise of all participants (Desimone & Hill, 2017) which contributes to district wide improvement (Darling-Hammond et al., 2017). Collegial collaboration, along with increasing educator effectiveness, positively impacts teacher agency (Calvert, 2016), contributing to a cycle of improvement (Clarke & Hollingsworth, 2002). In an analysis of high-performing systems, Jensen et al. (2016) note that teacher change is more likely to result from collaboration with colleagues than from facilitator directed instruction. The mediating effects of teacher beliefs (Guskey, 2002) and teacher content knowledge (Covay-Minor, Desimone, Lee, & Hochberg, 2016) on changes in teacher practice necessitate the inclusion of collaborative opportunities in professional learning designs.

Collaboration embodies sociocultural theory. Through collaboration, teachers share and construct knowledge regarding content, instructional practices, and beliefs (Spillane et al., 2002). The critical dialogue that occurs during formal and informal collaboration challenges educators to consider alternative perspectives (Mezirow, 1978) and serves as a source of scaffolding (Vygotsky, 1978) in the development of teacher expertise. In secondary education settings, collaborative groups act as learning communities through which the development of subject-specific content and pedagogical content knowledge occurs (Fisher & Frey, 2014). Learning communities foster growth and contribute to the sustainability of effective instructional practices (Learning Forward, 2011).

Supportive

Effective professional learning culminates in improved student outcomes (Learning Forward, 2011). Implementation bridges educator and student learning (Desimone & Garet, 2015), a bridge that is strengthened through facilitator support (Jensen et al., 2016; Learning Forward, 2011). Garet et al. (2001) recognize that opportunities for teachers to observe experts in modeling situations or participate in classroom coaching situations support the development of effective instructional practices. Instructional coaching and modeling provide opportunities for experimentation and reflection, activities crucial to instructional development (Clarke & Hollingsworth, 2002). Systems that create opportunities for teachers to observe and co-teach with colleagues, experience increased learning gains for teachers and students (Jensen et al., 2016).

Situated PD aligns with the premises of sociocultural theory and requires a degree of support (Polly & Hannafin, 2011). Garet et al. (2001) recognize that opportunities for teachers to observe experts in modeling situations or participate in classroom coaching situations supports the development of effective instructional practices. Modeling and coaching are a source of scaffolding as teachers transfer learned strategies into classroom practices, an application of Vygotsky's zone of proximal development for teachers (Polly & Hannafin, 2011). Empirical research supports the use of either "side-by-side or supervisory coaching" (Kretlow & Bartholomew, 2010, p. 294) for increasing the implementation of learned instructional strategies. Darling-Hammond et al. (2017) agree that expert scaffolding, in the form of instructional modeling, sustains the implementation of new curriculum or new instructional strategies. Aside from providing instructional support, modeling provides participants with a "clear vision of what best practices look like" (Darling-Hammond et al., 2017, p. v). A transformation from traditional to reform-minded mathematics instruction is supported when teachers are provided this vision through coaching and modeling activities.

Summary of Intervention Literature

Interventions that focus on building procedural and conceptual knowledge assist learners in developing a firm understanding of mathematics. The use of manipulatives and multiple representations help students bridge the gap between concrete and abstract reasoning (Hughes et al., 2014; Larbi & Mavis, 2016; Witzel et al., 2003). Curriculum that includes an analysis of worked out examples supports working memory and assists learners in developing procedural fluency (Booth et al., 2015, Booth et al., 2013; Lynch & Star, 2013). Interventions that promote self-explanation and mathematic discourse support mathematical development (Hudesman et al., 2013; Hutchinson, 1993; McEldoon et al., 2013; Stein et al., 2008). Besides providing support for conceptual understanding, many of these interventions assist in the identification and correction of students' misconceptions (Booth et al., 2015; Hutchinson, 1993; Witzel et al., 2003).

Interventions which reform the instructional practices of teachers also positively affect student understanding of mathematics. Reform-minded instructional practices align with recommendations by the National Mathematics Advisory Panel (2008) and NCTM (2014) and lead to increased student achievement in mathematics (Jong et al.,2010; Krupa & Confrey, Lawson et al., 2012). PD programs successfully transform instructional practices, provided they include the elements of education, support, and time for practice and reflection (Adamson et al., 2003; Borko et al., 2005; Loucks-Horsley et al., 2003). PD for teachers also leads to learning gains for students (Adamson et al., 2003). PD would be an effective intervention for transforming the traditional pedagogical methods of the mathematics teachers of this professional context.

Proposed Intervention for the Professional Context

The purpose of the proposed intervention is to increase the prevalence of reform-minded instruction for the mathematics teachers of this professional context. An apprenticeship model of PD, which includes elements found to be effective in the intervention literature, will serve as the primary intervention for the mathematics teachers at this high school. Teachers will engage in awareness and knowledge building activities (Adamson et al., 2003; Borasi et al., 1999; Borko et al., 2005; Loucks-Horsley et al., 2003), practice implementing reform-minded mathematics lesson with their peers and students (Borasi et al., 1999; Loucks-Horsley et al., 2003), and spend time reflecting on their learning with colleagues (Borko et al., Loucks-Horsley et al., 2003).

The awareness and knowledge building activities will expose teachers to student-centered instructional strategies designed to increase mathematical discourse, develop conceptual knowledge, and engage students in collaborative activities. Specific strategies include analyzing worked out examples, using prompting to increase self-explanation, and using manipulative to represent abstract concepts, all which are proven effective in the intervention literature (Booth et al. 2015; Larbi & Mavis, 2016; McEldoon et al., 2013).

The intended intervention provides support for teachers as they develop expertise in implementing new instructional strategies. Much like the scaffolding required for students as they develop cognition (Vygotsky, 1978), the proposed intervention assists teachers as they transform their pedagogy and increase their RTOP score.

Chapter 4: Intervention Procedure and Program Evaluation Methodology

Situated Apprenticeship Professional Development for Mathematics Educators

The empirical literature establishes a clear connection between reform-minded instructional practices and student achievement in mathematics (Boaler & Staples, 2008; Jong et al., 2010; Holt et al., 2015; Piburn & Sawada, 2000). Reform-minded instructional practices include pedagogy that is student-centered and encourages conceptual understanding, reasoning, and collaboration (Eddy et al., 2015; NCTM, 2014). Compared with traditional instructional practices, lessons that engage students and require interaction between learners result in greater student learning gains (Hake, 1998). Furthermore, instruction that provides students an opportunity to integrate knowledge and form connections between isolated skills and broad concepts stimulates cognitive development (Bruer, 2008; Hardiman, 2012).

Based on the data from the needs assessment, discussed in Chapter 2, and the empirical literature regarding the benefits of reform-minded instruction, outlined in Chapter 3, an intervention targeting a transformation of mathematics instruction through professional development (PD) was proposed. As suggested by Desimone (2009), Loucks-Horsley et al. (2003), and Saderholm et al. (2017) PD that aims to transform instructional practices is grounded in the needs of the local context. Needs assessment data revealed that most (63.5%) mathematics teachers desire training on how to implement this state's Mathematical Practice Standards. Needs assessment data also indicated that the mathematics teachers at this high school would benefit from training to increase their knowledge of reform-minded pedagogy and provide them with strategies for implementing reform-minded instruction. Aligning with a sociocultural perspective and the tenets of reform-minded instruction for students, a situated apprenticeship professional development model (SAPD) was developed and implemented. The SAPD engaged educators in authentic learning tasks to develop and refine content and pedagogical knowledge (Polly & Hannafin, 2011; Prather & Brissenden, 2008; Webster-Wright, 2009).

Purpose of the Study

The purpose of this study was to investigate the effects of a SAPD program on high school mathematics teachers' knowledge of reform-minded instruction, beliefs regarding reform-minded instruction, and implementation of reform-minded instructional practices. This study employed a convergent mixed method (QUAN + QUAL) design (Creswell & Plano Clark, 2011). The convergent design allowed for the simultaneous collection of quantitative and qualitative data as independent strands of data. Subsequent data analysis involved the merging of quantitative and qualitative results and a discussion of convergent or divergent findings (Creswell & Plano Clark, 2011). This convergent mixed methods study was framed by the following research questions:

Outcome Research Questions:

RQ 1: How did high school mathematics teachers' knowledge of reform-minded instructional practices change as a result of the situated apprenticeship professional development program?

RQ 2: How did high school mathematics teachers' pedagogical beliefs change as a results of the situated apprenticeship professional development focused on reform-minded instruction?

RQ 3: How did high school mathematics teachers' use of reform-minded instructional practices change as a result of the situated apprenticeship professional development program?

Process Research Questions:

RQ 4: What aspects of the situated apprenticeship professional development program do high school mathematics teachers describe as most meaningful in their transformation toward reform-minded instruction?

RQ 5: What aspects of the situated apprenticeship professional development program do high school mathematics teachers describe as least meaningful in their transformation toward reform-minded instruction?

RQ 6: To what extent was the situated apprenticeship professional development program implemented as designed?

Research confirms that PD is essential for instructional reform (Desimone, 2009; Garet et al., 2001), the goal of the SAPD program. Specific to reform-minded instruction, training on reform-minded instruction, collaborative participation and reflection, and instructional support are associated with an increase in knowledge and an increase in implementation of learned strategies (Amrein-Beardsley & Popp, 2012; Boston, Bostic, Lesseig, & Sherman, 2015;

MacIsaac & Falconer, 2002). A theory of treatment (see Figure 4.1) makes visible the mechanisms by which the planned inputs become the desired outputs (Leviton & Lipsey, 2007). As discussed in Chapter 2, prevalence of traditional instruction throughout the mathematics department of this researcher's context provides a possible explanation for current low levels of achievement in mathematics based on student performance on end-of-course exams. Adapted from Desimone's (2009) framework for effective PD, the theory of treatment is backed by research which supports a causal chain (Leviton and Lipsey, 2007) linking PD components to changes in content and pedagogical content knowledge (Covay-Minor et al., 2016), changes in teacher beliefs (Roehrig and Kruse, 2005) and a transformation of pedagogy (Glazer & Hannafin, 2006; Polly & Hannafin, 2011). Although empirical support provides evidence for ruling in the theory of treatment as a plausible explanation (Leviton & Lipsey, 2007), external variables such as participant responsiveness intervene. A successful treatment results in outcomes ranging from minimal to maximum magnitude (Leviton & Lipsey, 2007). At a minimum, the SAPD program intended to affect teachers' knowledge of reform-minded pedagogy, a hallmark of effective PD (Garet et al., 2001). The desired maximum effect would be an increase in student achievement. Measurable outcomes connecting these extremes consist of changes in teachers' knowledge of reform-minded instruction, beliefs regarding reform-minded instruction, and use of reform-minded instructional practices.

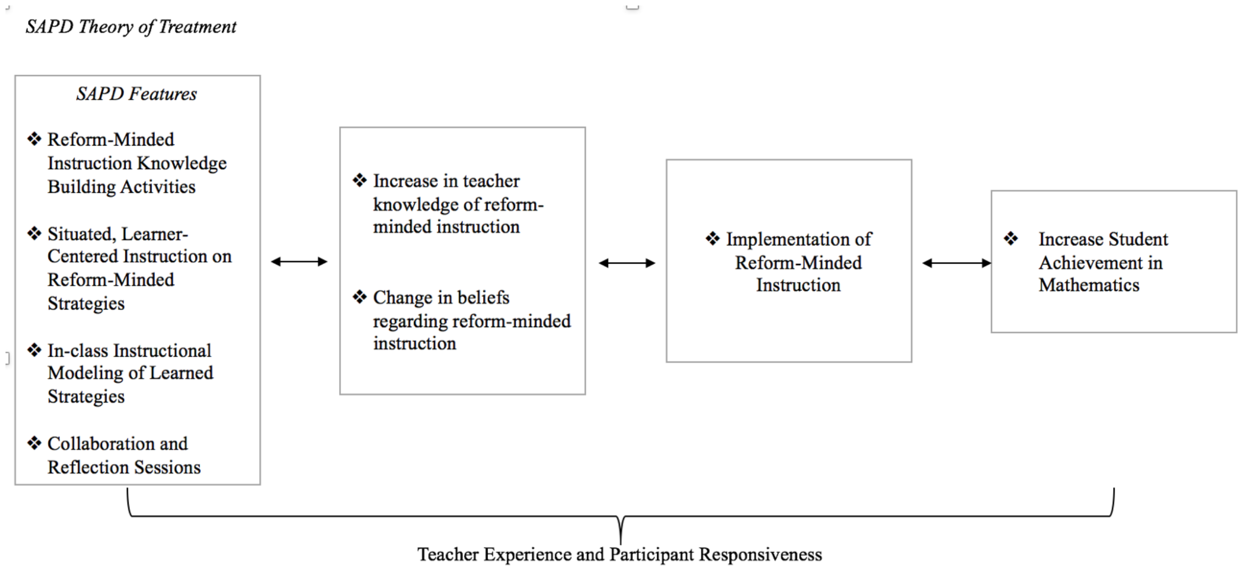

Figure 4. 1. Description of the theory of treatment that proposes a transformation of instructional practices as a result of the SAPD intervention.

Method

Participants and Recruitment

A year-long SAPD intervention was designed with the understanding that professional development exists to promote educators' instructional effectiveness (Avalos, 2011; Jensen et al., 2016). While increasing student achievement in mathematics remain a distal outcome in the theory of treatment (Figure 4.1), the intervention aimed to impact the knowledge, beliefs, and instructional practices of the mathematics educators of a Midwestern, public high school. Participants were obtained through purposive sampling (Martinson & O'Brien, 2010) dictated by eligibility criteria (Cook, Godiwalla, Brooks, Powers, & John, 2010), namely that participants were state certified to teach high school level mathematics and were currently teaching in the Midwestern high school of interest. All thirteen mathematics teachers at this Midwestern high school were eligible to participate. Participants were recruited during a face-to-face meeting prior to the start of the 2018-19 school year. The population consisted of eight mathematics content teachers, four mathematics intervention specialists who serve in a co-teaching capacity, and one intervention specialists who taught in a self-contained classroom. The intervention was designed to take place during contractual time for teachers, increasing the likelihood of recruiting and retaining all mathematics teachers at this high school.

All eligible participants (N =13) attended a 15-minute recruitment meeting in August of 2018. The recruitment meeting was conducted by the school librarian, a colleague of the researcher, in an attempt to reduce possible feelings of coercion on the part of potential participants. The school librarian read a recruitment script and distributed participant consent forms. Teachers returned consent forms in sealed envelopes to the researcher's school mailbox. Teachers who provided consent to participate were assigned a numeric pseudonym to preserve anonymity. A total of 10 mathematic teachers agreed to participate in this study. Teachers who chose not to participate did not provide specific reasoning. During the year-long study, one participant took a leave of absence from school and did not complete the study. The final number of participants for this study was nine high school mathematics teachers.

The nine participants in this study were state certified in their respective content areas and all were full-time district employees. The majority of participants (77.8%; N = 9) had over 15 years of teaching experience. Of those with over 15 years of experience, four teachers had between 25 and 29 years of experience as mathematics teachers. The remaining two participants had 4 and 5 years of experience respectively. Most participants (66.6%; N = 9) held master's degrees in education and an additional two participants were currently working on completing graduate level degrees. When asked to consider recent participant in professional development programs, seven participants indicated that they have attended PD programs targeting mathematics content and pedagogical content within the past 24 months.

Statistical power, the probability of accurately determining statistical significance, decreases with smaller sample sizes (Lipsey & Hurley, 2009). The sample size ($N = 9$) was a predominant concern for this study. As indicated by a G*Power analysis, a sample size of nine reduces statistical power to achieve the desired effect size to .24. Reducing the power to .24 increases the risk for Type I and Type II errors, the probability of drawing incorrect statistical conclusions (Lipsey & Hurley, 2009). The small sample size ($N = 9$) for this study reduced the power to detect a causal relationship and violated statistical tests assumptions, both threats to statistical conclusion validity (Shadish, Cook, & Campbell, 2012). Using reliable outcome measures and strengthening treatment implementation strengthens both power and statistical conclusion validity (Shadish et al., 2012). The RTOP (Sawada et al., 2002) and the Mathematics Teaching Pedagogical and Discourse Beliefs Instrument (Lischka & Garner, 2016) report reliability ratings of .97 and .96 respectively indicating high reliability. Furthermore, reliable treatment delivery was expected given that the intervention occurred during contractual hours for teachers and that researcher was the sole intervention provider.

Instruments and Measures

Reform-minded instruction knowledge surveys. Knowledge of reform-minded instructional practices was a proximal variable of interest in this study. PD programs that concentrate on developing teachers' pedagogical content knowledge lead to the implementation of effective instructional practices (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Garet et al., 2001). Furthermore, pedagogical content knowledge building activities contribute to the success of PD programs that seek reform mathematics instruction (Ebert-May et al., 2011; Hill et al., 2005).

SAPD session exit tickets measured participants knowledge of reform-minded instructional practices. Each exit ticket ($N = 5$) consisted of five multiple-choice items and one open-ended items. The researcher-created exit tickets contained one multiple-choice item for each of the RTOP subscales; lesson design and implementation, propositional pedagogical knowledge, procedural pedagogical knowledge, communicative interactions, and student-teacher relationships (Sawada et al., 2002). Exit ticket items asked participants to describe reform-minded instructors, identify behaviors of students in reform-minded classrooms, and recognize characteristics of reform-minded lessons. Content for the multiple-choice questions and corresponding possible answers came from the "Classroom Observation Project" (Creating a Student-Centered Classroom, 2016). The single open-ended item on each exit ticket was, "What questions or comments do you have at this time regarding reform-minded instruction?" Appendix C contains the five SAPD session exit tickets.

Teacher beliefs instrument. Teacher beliefs regarding reform-minded instruction was a second variable of interest in this study. Research indicates that educator beliefs greatly influence educators' decisions regarding instructional practices (Campbell et al., 2014; Desimone, 2009; Gamoran et al., 1997; Guskey, 2002; Maccini & Gagnon, 2002). The Mathematics Teaching Pedagogical and Discourse Beliefs instrument (MTPDBI) is a 20-item multiple-choice survey created to measure the beliefs and dispositions that mathematics teachers hold regarding reform-minded instruction (Lischka & Garner, 2016). The MTPDBI categorizes teacher beliefs into four domains: (a) learner-centered, (b) approaching, (c) emerging, and (d) teacher-centered (Lischka & Garner, 2016). Table 4.1 illustrates belief statements that correspond with the four MTPDBI domains. For each of the 20 items, teacher responses are coded 1 to 4 where a score of 4 indicates a learner-centered belief and a score of 1 represents a teacher-centered belief. Composite scores are used to compare teacher beliefs along a teacher-centered learner-centered continuum. PD facilitators find the data from this survey useful for stimulating discussions on reform-minded instruction (Lischka & Garner, 2016). The complete MTPDBI can be found in Appendix D.

The development of the MTPDBI included analyzing Wright maps, displays of Rasch person and item measure data. Wright maps illustrate Rasch measures and allow the developers of instruments to assess the degree to which respondent's answers align with what is expected given the degree of difficulty of the survey items (Boone, 2016). Authors report an item separation score of 5.16 which indicates that the MTPDBI items appropriately differentiate between domains (Lischka & Garner, 2016). An item separation score of .96 suggests a high degree of consistency in regard to the placement of items on the scale. Authors conducted qualitative research to determine the effectiveness of the MTPDBI in distinguishing between teachers who held traditional beliefs and those who held reform-minded beliefs. Lischka and Garner (2016) attribute a lower person reliability score (.71) to the small sample size (n=187) in the pilot study. Follow-up qualitative studies, conducted by the authors indicate that participant scores on the MTPDBI align with observed instructional practices.

Table 4. 1

Mathematics Teaching Pedagogical and Discourse Beliefs Instrument Domains

MTPDBI Domain	Corresponding Belief Statements
Learner-Centered	• Mathematics is a dynamic human creation that is a cultural product. • Learning mathematics requires exploration and personal discovery. • The role of the teacher is a facilitator and questioner. Teachers pose problems and encourage student exploration. • Teacher as co-teacher and co-learner. Teacher monitors discussions while fully engaged but allows students to direct learning. Teacher employs hermeneutic listening.
Approaching	• Mathematical ideas are connected in meaningful ways that can be discovered. • Learning mathematics involves active construction of ideas related to prior knowledge. • Teachers are explainers of reasoning and learning includes understanding reasoning that supports procedures. • Teacher facilitates student-student talk and employs interpretive listening. Student ideas are explored.
Emerging	• Mathematics is a collection of unified and connected facts. • Learning mathematics requires acquisition of knowledge as well as procedural skill. • The role of the teacher is to instruct in proper procedures. • The teacher is the only questioner with student ideas left unexplored. Students do not talk with each other. Teacher employs evaluative listening with some effort for follow-up questions.
Teacher-Centered	• Mathematics is a collection of facts, rules, and skills. • Learning mathematics requires mastery of skills and memorization of rules. • The role of the teacher is to direct all learning. Students are compliant. • True discourse is not present. Communication is done primarily by the teacher and listening is not a concern of the teacher.

Source: Taken from Lischka & Garner, 2016.

Reformed Teaching Observation Protocol (RTOP). The third variable of interest in this study was the level at which participants implemented reform-minded instructional practices. Reform-minded mathematics instruction positively impacts student achievement in mathematics (Boaler & Staples, 2008; Jong et al., 2010; McCaffrey et al., 200; Sawada et al., 2002; Smith et al., 2015). The RTOP consists of 25 Likert-scale items divided into five subscales, each representing certain aspects of reform-minded instruction: (a) lesson design and implementation, (b) propositional pedagogic knowledge, (c) procedural pedagogic knowledge, (d) communicative interactions, and (e) student-teacher relationships (Sawada et al., 2002). Researchers created the RTOP in response to a need for measuring mathematics and science instruction following the reform movement in education (Piburn & Sawada, 2000). The RTOP intends to measure the use of reform-minded teaching practices in mathematics, practices that are aligned to rigorous standards, problem-solving oriented, collaborative, and supported by research (Lawson et al., 2002; Piburn & Sawada, 2000). As indicated in Table 4.2, five categories of RTOP scores denote the level of reform-minded instruction present in a lesson (Ebert-May et al., 2011). The complete RTOP protocol can be found in Appendix B.

Reliability and validity of the RTOP have been established through reports of interrater reliability (98%) (Sawada et al., 2002), face validity (Sawada et al., 2002), and predictive reliability (Ellis, Malloy, Meece, & Sylvester, 2007). The creators of the RTOP report a Cronbach's alpha score of .97 which confirms the validity of the RTOP as a measure of reform-minded instruction (Sawada et al., 2002). Research supports a significant, positive correlation (r=.94, p<.001) between RTOP scores student performance in mathematics (Lawson et al., 2002). Furthermore, reform-minded instruction, reflected by high RTOP scores, enhances students learning and is connected to positive learning outcomes (Jong et al., 2010; Sawada et al., 2002).

Table 4. 2

Categorization of Reformed Teaching Observation Protocol Scores (Ebert-May et al., 2011)

RTOP Level	RTOP Score Range	Instructional Practices
I	0-30	Lecture Oriented Teacher as primary source of instruction
II	31-45	Lecture accompanied by demonstration and minor student participation Teacher as primary source of instruction
III	46-60	Hands-on student engagement Students involved in carrying out tasks
IV	61-75	Active student engagement in delivery of lesson Students involved in carrying out tasks
V	76-100	Active student engagement in creation and delivery of lesson Lesson informed by student inquiry and reflection

Panorama survey. The effectiveness of PD programs at inducing change depends on levels of participant engagement and responsiveness (Calvert, 2016; Polly & Hannafin, 2011). The Panorama Teacher Survey consists of a collection of instruments designed to collect data from instructional staff on 18 topics (Panorama Education, 2015). The full survey includes instruments designed to measure teachers' perceptions of (a) professional development and (b) feedback and coaching. The professional learning instrument consists of eight Likert scale items and the feedback and coaching instrument consists of five Likert scale items that ask teachers to rate the amount, quality, and usefulness of professional development and feedback offerings (Panorama Education, 2015). The individual instruments are designed to be customized dependent upon contextual needs. Examples of customized survey items are presented in Table 4.3 and Table 4.4. The complete survey used in this study can be found in Appendix E.

Panorama Education (2015) partners minimized measurement error when developing survey item by: (a) including items are questions rather than statements, (b) excluding agree-disagree response items, (c) avoiding double-barreled items, (d) providing five response options, and (e) using verbal rather than numeric labels (Panorama Education, 2015). These steps align with recommendations for question formula (O'Leary, 2104) and contribute to the validity of the survey. Panorama researchers recognize the need for continued pilot testing in determining reliability and validity data on the Panorama Teacher Survey (Panorama Education, 2015). However, the Panorama Student Survey, designed using the same techniques, received Cronbach's alpha scores of α=.70 or greater on each of the subscales (English, Burniske, Meibaum, & Lachlan-Hache, 2016), confirming the quality of Panorama surveys for use in the field of education.

Table 4. 3

Customized Panorama Professional Learning Survey Items

Original Item Wording	Customized Item Wording
How relevant have your professional development opportunities been to the content that you teach?	How relevant has this SAPD opportunity been to the content that you teach?
Through working at your school, how many new teaching strategies have you learned?	Through participation in this SAPD, how many new teaching strategies have you learned?
Overall, how much do you learn about teaching from the leaders at your school?	Overall, how much did you learn about teaching from this SAPD?

Table 4. 4

Customized Panorama Feedback and Modeling Survey Items

Original Item Wording	Customized Item Wording
How much feedback do you receive on your teaching?	How much feedback did you receive on your teaching during the SAPD program?
How useful do you find the feedback you receive on your teaching?	How useful did you find the classroom modeling sessions?
At your school, how thorough is the feedback you receive in covering all aspects of your role as a teacher?	During this SAPD program, how thorough was the feedback you received in covering all aspects of the implementation of reform-minded instruction?

Semi-structured interviews. The semi-structured interview items were informed by the research questions guiding this study. The interview protocol (see Appendix F) consists of 10 open-ended items designed to (a) assess participants' overall perceptions of the SAPD program, (b) identify SAPD program components that participants described as contributing or failing to contribute to their professional growth, and (c) appraise participants' views regarding changes to the SAPD program. The interview protocol culminated by inquiring "Is there anything else you would like to share regarding your experiences as a participant in the SAPD program?"

Program artifacts. Program artifacts provided qualitative data informing changes in participants' knowledge, beliefs, and instructional practices. Program artifacts included audio recordings of SAPD sessions, hand-written SAPD session activity artifacts, and participant implementation feedback forms. Qualitative research relies on an examination of artifacts to determine emergent themes or codes (Bayar, 2014; Luft & Roehrig, 2007). Themes which align with the category ratings of the RTOP (see Table 4.2) and the MTPDBI (see Table 4.1) informed the researcher of changes in pedagogical knowledge, beliefs, and instructional practices.

Member checking with the study participants to ensure accuracy of responses established the credibility of this measure (Creswell & Plano Clark, 2011).

Attendance logs. Participant attendance data, recorded on attendance logs, served as an indicator of dose received; an element of program implementation (Linnan & Steckler, 2002). Active participation, collaboration, and reflection improve program efforts (Desimone, 2009) and attendance predicates these collaboration and reflection activities as outlined in the logic model. Attendance serves as the foundation of the theory of change in that teachers must be present to receive the treatment.

Pre-Intervention Survey. A pre-intervention teacher survey (see Appendix G), created from demographic and professional development items from the Teaching and Learning International Survey (TALIS) (OECD, 2010), measured indicators such as (a) teacher certification, (b) teaching experience, and (c) professional development experience. These contextual factors have potential moderating effects on successful program implementation and impact program outcomes (Baranowski & Stables, 2000). Forty-five countries collaborated on TALIS items, under the direction of the Organization for Economic Co-operation and Development (OECD, 2010), informing the reliability and validity of the survey.

Procedure

The following section provides a description of the components of the SAPD intervention, the implementation timeline, the data collection process, and the plan for data analysis.

SAPD Program Intervention

In response to extant literature outlining the components of effective professional development (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Garet et al., 2001) and informed by a sociocultural approach to learning (Gavelek & Raphael, 2014; Vygotsky, 1978), and SAPD program was designed. The SAPD program included (a) face-to-face instructional sessions that engaged teachers in reform-minded instructional strategies, (b) individual classroom modeling sessions that provided opportunities for participants to observe the implementation of reform-minded instruction, and (c) additional face-to-face sessions that provided time for critical reflection and collegial collaboration. The hypothesis was that high school mathematics teachers who participate in the SAPD program would strengthen their understanding of reform-minded instruction, would experience a change in pedagogical beliefs, and would increase the use of reform-minded instructional practices in their respective classrooms.

The SAPD program combined elements of situated learning (Clarke & Hollingsworth, 2001; Prather & Brissenden, 2008), embedded teacher assistance (Stein et al., 1999), and instructional modeling (Desimone & Pak, 2017). Research promotes PD that occurs in context of the learning environment and involves collaboration from teachers within the same school, department, or grade level (Garet et al., 2001; Stein et al., 1999). As shown in Table 4.5, the SAPD program consisted of training teachers on reform-minded instructional strategies, providing teachers the opportunity to collaborate on planning reform-minded lessons, allowing teachers to observe reform-minded instruction through instructional modeling, and leading teachers in a reflection of their experiences. The design of the SAPD program included both private and public space for participants to develop their instructional expertise (Raphael et al., 2014). The SAPD program intended to provide participants with a total of 21 hours of PD over the course of the 2018-19 school year.

Table 4. 5

Description and Duration of Planned Intervention Components

Activity	Timeline	Duration	Description
Face-to-face learner-centered instructional sessions	Late September 2018 Early October 2018	One hour per session 3 total sessions	Participants view video-taped lessons representing examples of reform-minded and traditional instruction Participants discuss possible ratings for each lesson using RTOP
Face-to-face, learner-centered instructional sessions	October 2018 - March 2019 (monthly)	One hour per session 6 total sessions	Participants engage as learners to experience reform-minded instructional strategies Participants collaborate in subject-level teams to plan lessons utilizing learned strategies
Individual classroom modeling sessions	October 2018 - March 2019 (monthly)	45 minutes 1 class period per participant 6 total periods per participant	Researcher models use of a reform-minded instructional strategy

Table 4.5 (continued)

Face-to-face collaboration and reflection sessions	October 2018 – March 2019 (monthly)	45 minutes per session 6 total sessions	Whole-group reflection/sharing of experiences with implementing reform-minded instructional strategy Participants collaborate in subject-level teams to plan lessons utilizing learned strategies
Personal experience sharing	October 2018 – March 2019 (monthly)	Varied Approximately 15 minutes per session	Participants share results of classroom experimentation of reform-minded instructional strategy

Training on reform-minded instructional practices. Instructional change stems from an increase in both content and pedagogical content knowledge (Garet et al., 2001; Guskey, 2003; Saderholm et al., 2017). PD that results in the adoption of reform-minded instruction includes knowledge building activities (Prather & Brissenden, 2008). The first component of the intervention consisted of training activities where teachers (a) analyzed videos of traditional and reform-minded mathematics lessons, (b) differentiated between characteristics of traditional and reform-minded mathematics lessons, (c) strategized steps to reforming mathematics instruction, and (d) engage as learners in reform-minded instructional strategies. Following the apprenticeship model of Prather and Brissenden (2008), all SAPD sessions provided teachers the opportunity to experience reform-minded instruction from the perspective of a learner. For example, each SAPD face-to-face session (a) accessed participants' prior knowledge of reform-minded instruction, (b) provided opportunities for participants to explore a specific characteristic of reform-minded instruction (e.g., student discourse, teacher as facilitator), and (c) was guided by participants' needs. The face-to-face sessions mimicked the style of reform-minded instruction (Sawada et al., 2002).

Following the 3 initial knowledge building sessions, participants engaged in a trio of activities designed to provide them with specific instructional strategies considered to be reform-minded. The trio of activities included (a) engaging as a learner with the instructional strategy, (b) observing as the research modeled the strategy in individual participants' classrooms, and (c) reflecting on attempts at implementing the strategy individually. Participants were briefly introduced to six reform-minded instructional strategies. The strategies include increasing student discourse with think-pair-share (Prather & Brissenden, 2008), encouraging student exploration through Desmos classroom activities (Desmos, 2019), promoting meaningful learning through three-act math tasks, developing mathematics communication through the use of language frames (Rumsey and Langrall, 2016), analyzing correct and incorrect examples through the application of AlgebraByExample lessons (Booth et al., 2015) and focusing on conceptual understanding with ConcepTests (MacIsaac & Falconer, 2012). Table 4.6 outlines a description of these activities and how they align with reform-minded practices. After reviewing the six instructional strategies, participants were asked to choose 3 strategies for which they would like in-depth training. Popular vote indicated that participants desired to learn more about Desmos classroom activities (Desmos, 2019), three-act math tasks, and AlgebraByExample (Booth et al., 2015). Having available reform-minded resources impacts the implementation of reform-minded instruction (Roehrig & Kruse, 2005). In addition, active learning experiences increase teacher knowledge and result in a change instructional practices (Stein et al., 1999).

Table 4. 6

Description of Reform-Minded Instructional Strategies

Instructional Strategy	Description	Elements of Reform-Minded Instruction
Desmos Classroom Activities	Digital, exploratory mathematical activities designed to stimulate mathematical inquiry	✓ Promotes reasoning and problem-solving ✓ Encourages student discourse ✓ Utilizes multiple representations ✓ Develops conceptual understanding
Three-Act Math Tasks	Instructional technique in which teachers pose meaningful problems in three stages: (1) pose problem to pique curiosity and motivate, (2) provide scaffolding as students explore resources that will assist in problem solving, and (3) resolve problem through review of student solutions	✓ Promotes inquiry and reasoning ✓ Elicits student thinking ✓ Promotes problem-solving ✓ Encourages application of concepts ✓ Facilitates student discourse
Think-Pair-Share	Peer discussion technique designed to develop critical thinking and reasoning through wait-time and discourse	✓ Promotes reasoning and problem-solving ✓ Increases metacognition and communication skills ✓ Develops conceptual understanding ✓ Utilizes meaningful questioning ✓ Elicits student thinking
Analyzing Worked Examples	Instructional technique in which students analyze correct and incorrect worked examples to develop procedural knowledge, formulate concepts, and clarify misconceptions	✓ Develops procedural knowledge through conceptual understanding ✓ Promotes inquiry and reasoning ✓ Encourages analysis to clarify misconceptions ✓ Provides opportunity to form connections ✓ Elicits student thinking

Table 4.6 (continued)

ConcepTest	Student response technique designed to assess progress toward learning goals through higher-order, conceptual-based multiple choice items	✓ Increases student discourse ✓ Develops conceptual understanding ✓ Elicits student thinking ✓ Supports productive struggle

Collegial collaboration and critical reflection. Research on effective PD promotes collaborative exchange and collective participation among teachers of common subjects, grade levels, or departments (Guskey, 2003). Collective participation not only increases knowledge (Garet et al., 2001; Stein et al., 1999), but serves as a motivator for teachers who find traditional PD irrelevant to their needs or context (Bayar, 2014; Stein et al., 1999). Embedded in the face-to-face SAPD sessions was time for teachers to meet with subject specific teams to collaborate on developing reform-minded lessons based on the knowledge and experience gained in training activities. During collaborative time, subject specific teams reviewed current lessons plans and adapted lessons to include reform-minded strategies. Each collaborative session lasted for approximately 20 to 30 minutes, half of the allotted time for each SAPD face-to-face session.

Along with a focus on knowledge building and collaboration, successful PD includes time for reflection (Garet et al., 2001; Garrett & Steinberg, 2015; Saderholm et al., 2017; Stein et al., 1999). As indicated in Table 4.5 the SAPD program includes time for individual and collaborative critical reflection, each an hour in duration. During each reflection session, participants discussed experiences in implementing reform-minded instructional strategies. During these face-to-face sessions, participants also met in subject specific teams to revise reform-minded lessons. Collaborative reflection promotes a culture of experimentation, examination, and reflection, all of which lead to improved teaching (Jong et al., 2010; Stein et al., 1999). Teachers report that peer feedback and advice, and collaborating with colleagues on instructional practices is critical to instructional improvement (Amrein-Beardsley & Popp, 2012; Jong et al., 2010). Furthermore, teachers desire the opportunity to examine and reflect when implementing changes in instruction (Stein et al., 1999).

Instructional modeling. An apprenticeship model of PD assists teachers in transferring gained knowledge into practice (Prather & Brissenden, 2008). Instructional modeling supports teachers as they gain experience and comfort in implementing instructional change (Roehrig & Kruse, 2005). The instructional coaching component consisted of the SAPD program facilitator modeling reform-minded lessons in the classroom context and providing feedback during and after the implementation. The SAPD program called for each participant to receive one instructional modeling session (45-minute class period) per month (6 sessions total). Embedding assistance in the classroom leads to successful implementation and sustained instructional change (Garet et al., 2001; Stein et al., 1999). PD that provides a knowledge base, but does not support teachers as they implement strategies, fails to result in a change in instructional practices (Ebert-May et al., 2011).

Data Collection

Following a convergent mixed methods design (Creswell & Plano Clark, 2011), quantitative and qualitative data was collected concurrently throughout the intervention. The quantitative and qualitative data was merged for both process and outcome evaluations. Sources of quantitative data included (a) pre- and post-intervention RTOP (Sawada et al., 2002) scores, (b) pre- and post-intervention MTPDBI (Lischka & Garner, 2016) scores, (c) pre-, mid-, and post-intervention exit ticket scores, (d) post-intervention Panorama survey (Panorama Education, 2015) scores, and (e) attendance logs. Sources of qualitative data included (a) SAPD session audio recordings, (b) SAPD session activity artifacts, (c) participant implementation feedback forms, and (d) post-intervention semi-structured interviews. The following section will outline the data collection process for each measure.

Outcome Evaluation

Outcome evaluation indicators were evaluated using a one-group pretest-posttest within-participants design (Shadish et al., 2002). The effectiveness of this intervention was determined by analyzing changes in the proximal outcomes of teacher knowledge, teacher beliefs, and mathematics instructional practices. The outcome evaluation plan included collecting quantitative pretest-posttest data for the distal outcome, implementation of reform-minded instructional practices, and the proximal outcome, teacher beliefs regarding reform-minded instruction. The outcome evaluation plan will also include collecting data from field notes, open-ended surveys, and participant presentations. Table 4.7 outlines the outcome data collection process.

Table 4. 7

Data Collection Matrix for Outcome Evaluation Plan

Indicator	Role of Indicator	Data Source(s)	Frequency	Responsibility
High school mathematics teachers' use of reform-minded instructional practices	Proximal Outcome Variable	Reformed Teaching Observation Protocol (Sawada et al., 2002)	Twice - pre-treatment and post-treatment	Intervention Implementer
		Semi-structured interviews	Once – upon completion of the intervention	
		Participant Implementation Feedback Forms	Monthly- during each reflective PD session	
High school mathematics teachers' beliefs regarding reform-minded instruction	Proximal Outcome Variable	Mathematics Teaching Pedagogical and Discourse Beliefs Instrument (Lischka & Garner, 2016)	Twice pre-treatment and post-treatment	Intervention Implementer
		Semi-structured interviews	Once –upon completion of the intervention	
		Participant implementation feedback forms	Monthly – during each reflective PD session	
High school mathematics teachers' knowledge of reform-minded instruction	Proximal Outcome Variable	SAPD session exit tickets	Five times – At intervals throughout the SAPD program	Intervention Implementer
		Semi-structured interviews	Once- upon completion of the SAPD program	
		Participant implementation feedback forms	Five times – at intervals throughout the SAPD program	

Knowledge surveys. Participants were asked to complete SAPD session exit tickets at five stages throughout the duration of the SAPD program. Exit tickets were administered electronically through SurveyMonkey. A link to the exit ticket was sent to participants' school email and participants were asked to complete the exit tickets prior to leaving the SAPD session. All participants received links to all SAPD session exit tickets. Non-responders were contacted personally and reminded to complete the respective exit ticket.

Beliefs inventory. Participants were asked to complete the MTPDBI (Lischka & Garner, 2016) pre- and post-intervention (September 2018 and April 2019). Paper copies of the instrument were hand-delivered to participants the week prior to the first scheduled observation and again during the week following the final SAPD session. Participants were given one week to complete the inventory and were asked to return the survey to the researcher's school mailbox. Non-responders were contacted personally and were provided an additional two days to complete the inventory.

Classroom observations. RTOP scores were collected prior to the onset of the intervention (September 2018) and again at the conclusion of the intervention (May 2019). The researcher assigned scores following classroom observations for each participant. The classroom observations were scheduled at the participants' convenience. To control for variation in instruction between class sections, the researcher observed the same class period for pre- and post-intervention observations.

Semi-structured interviews. Participants' perceptions of the SAPD program and SAPD program components were collected upon completion of the intervention (May 2019). Participants were interviewed one-on-one at a time and location determined by the participant.

Interviews were conducted orally and participants' responses were typed into a Microsoft Word document. Typed written responses were reviewed with each participant to ensure accuracy.

Program Artifacts. SAPD session were audio recorded and a verbatim transcript was typed upon the completion of the SAPD program. SAPD session hand-written artifacts (e.g., participant lesson plans, responses to activities) were collected throughout the intervention. Participant implementation feedback forms were shared with the researcher via Google Slides at five intervals throughout the SAPD program. Researcher field notes were taken throughout the duration of the SAPD program and were recorded on the researcher's personal, password protected laptop computer.

Process Evaluation

Process evaluation seeks to analyze how program outcomes result from program outputs (Linnan & Steckler, 2002). Process evaluation indicators assisted in analyzing the individual program components that contributed to the effectiveness of the intervention (Baranowski & Stables, 2000). The effectiveness of the intervention depended on a measure of fidelity describing the degree to which participants are engaged in and transformed by the program activities, defined by Dusenbury et al. (2003) as participant responsiveness. Active participation is critical to effective PD (Polly & Hannafin, 2011) especially when a change in instruction is involved (Guskey, 1986).

Process evaluation indicators included measures of participant responsiveness and program implementation. The success of this intervention depended on active engagement on the part of the teachers (Garet et al., 2001) and was likely impacted by participants' knowledge of reform-minded instruction and participants' pedagogical beliefs (Roehrig & Kruse, 2005). Therefore, participant responsiveness and program implementation were critical evaluation

Table 4. 8

Process Evaluation Data Collection Matrix

Process Evaluation Indicator	Role of Indicator	Data Collection Tool	Frequency	Responsibility
Delivery of 15 (one-hour) face-to-face SAPD sessions	Dose delivered	Attendance Log	Bimonthly –at each SAPD session	SAPD provider
Delivery of six instructional modeling sessions per teacher	Dose delivered	Researcher calendar	Monthly – at the end of each instructional modeling session	SAPD provider
Participants' attendance at 15 (one-hour) SAPD session	Dose received	Attendance logs	Bimonthly - at the end of each PD session	SAPD provider and participants
Participants' participation in 6 (45-minute) instructional modeling sessions	Dose received	Researcher calendar	Monthly – at the end of each instructional modeling session	SAPD provider and participants
Participants' perception of SAPD program	Dose received	Panorama Survey (Panorama Education, 2015)	Once – at the conclusion of the intervention	SAPD provider and participants
	Participant responsiveness	Semi-structured interviews	Once – at the conclusion of the intervention	
Participants' enactment of learned strategies	Participant responsiveness	Google Slide submissions	Monthly – during the reflection professional development session	SAPD provider and participants
	SAPD session audio recordings	SAPD session audio recordings	Bi-monthly – at each face-to-face SAPD session	

Table 4.8 (continued)

Participants' ratings of intervention components	Participant responsiveness	Panorama Survey (Panorama Education, 2015)	Once – upon completion of the intervention	SAPD provider and participants
		Semi-structured interviews	Bi-monthly – at each face-to-face SAPD session	

components. Table 4.8 outlines the process evaluation data collection process for this intervention.

Participant Perception Surveys. Teacher ratings of the SAPD program were collected once upon completion of the intervention (May 2019). Paper copies of the Panorama survey (Panorama Education, 2015) were hand-delivered to participants during the week following the final SAPD session. Participants were given one week to complete the survey and were asked to return the survey to the researcher's work mailbox. Non-responders were contacted personally and were provided an additional two days to complete the survey.

Semi-structured interviews. Data informing participants' perception of the SAPD program and the SAPD program components were collected during semi-structured interviews. One-on-one oral interviews were conducted at the conclusion of the SAPD program (May 2019). Participants' oral responses were transcribed into a Microsoft Word document and were reviewed with each participant to ensure accuracy.

Participant implementation feedback forms. Implemented feedback forms were collected monthly throughout the intervention. Participants were asked to fill-in a Google Slide template after implementing a reform-minded lesson with their classes. The template included spaces for participants to share the nature of the lesson, teacher feedback, student feedback, and further questions or concerns.

Program Artifacts. Program artifacts informing the process evaluation included SAPD session audio recordings, SAPD session hand-written artifacts (e.g., collaborative lesson plans, participant responses to SAPD activities), and researcher filed notes. Program artifacts were collected bi-monthly at each SAPD instructional and reflection session. Type-written researcher field notes were prepared following SAPD and instructional modeling sessions.

Attendance Logs. Attendance data for SAPD sessions were collected bi-monthly at the beginning of each of the 15 SAPD face-to-face session and recorded on attendance logs. Attendance data for the six SAPD instructional modeling sessions were collected at the beginning of instructional modeling session and recorded on the researcher's calendar.

Data Analysis

This study employed a convergent parallel mixed-methods design which allowed for the simultaneous analysis of quantitative and qualitative data (Creswell & Plano Clark, 2011). The convergent parallel mixed methods approach combined qualitative data, obtained from the SAPD session audio recordings, SAPD program artifacts, participant implementation feedback forms, and participants' responses to semi-structured interviews, with quantitative results, obtained from SAPD session exit tickets, MTPDBI surveys, and RTOP scores. As outlined by the summary matrix (Appendix H), the data analysis process included a quantitative analysis using descriptive and inferential statistics, and a qualitative analysis using a directed approach to content analysis (Hsieh & Shannon, 2005) to reveal a prior and emergent codes and themes.

Quantitative data from observation protocols, surveys, and exit tickets was entered into SPSS for analysis. Descriptive statistics revealed general trends in participants knowledge of reform-minded instruction, beliefs regarding reform-minded instruction, and use of reform-minded instructional practices prior to and at the conclusion of the intervention. Descriptive statistics also revealed trends in participants' perceptions of the SAPD program and program components.

Differences between pre-intervention and post-intervention scores for the outcome variables of beliefs regarding reform-minded instruction and implementation of reform-minded instructional practices were analyzed using the non-parametric Wilcoxon Signed Ranks test. A one-group pretest-posttest design was the most viable option for this intervention due to validity concerns, logistic concerns, and limited researcher resources. The within-participants design addressed the challenge of group differences that exists in comparison group designs (Henry, 2010). In this design, pretest scores served as an indicator of the measured outcome for the control group and posttest scores serve as an indicator for the treatment group (Stuart, 2007). In this study, each participant served as their own control (Henry, 2010).

Qualitative data from SAPD session audio recordings, SAPD session artifacts, participants' implementation feedback forms, and semi-structured interviews were analyzed using a directed approach to content analysis (Hsieh & Shannon, 2005). RTOP (Sawada et al., 2002) and MTPDBI (Lischka & Garner, 2016) subscales provided the themes under which qualitative data were organized. Qualitative data was first coded using a provisional coding method (Miles, Huberman, & Saldaña, 2014) where RTOP (Sawada et al., 2002) and MTPDBI (Lischka & Garner, 2016) subscale descriptors served as a priori codes. During the first phase of analysis, emergent codes were also noted and organized under themes. The second phase of analysis reviewed and confirmed provisional codes and their respective assignment to predetermined themes. The use of a directed approach to qualitative analysis involved possible bias (Hsieh & Shannon, 2005). Aligning codes to predetermined themes in theory could result in the over representation of desired results (Hsieh & Shannon, 2005). The use of an external reviewer increases the accuracy of findings (Hsieh & Shannon, 2005). The final stage of qualitative analysis involved asking an external reviewer, a Johns Hopkins University Doctor of Education graduate, to provide an individual analysis of qualitative data. The external reviewer and researcher compared assigned codes and themes which were in agreement with one another.

The small sample size used in this study ($N = 9$) threatened the statistical conclusion validity (Shadish et al., 2012) of the non-parametric Wilcoxon Signed Ranks test. Data analysis relied heavily on descriptive and qualitative data, justifying the choice of a mixed-methods approach. Qualitative data enhanced quantitative results revealing trends (Bamberger, Tarsilla, & Hesse-Biber, 2016) providing a more cohesive understanding participants' experiences in this study.

Chapter 5: Findings and Discussion

Effective professional development programs create authentic opportunities to learn including (a) situating the learning in context (Learning Forward, 2011; Raphael et al., 2014), (b) providing opportunities for collaboration (Desimone, 2009; Garet et al., 2001), (c) promoting a learner-centered approach (Darling-Hammond et al., 2017; Learning Forward, 2011), and (d) supporting teachers in their efforts to improve their teaching (Calvert, 2016; Darling-Hammond et al., 2017; Jensen et al., 2016). In fact, adult learning theorists purport that learning is enhanced when accompanied by social interaction (Raphael, Vasquez, Fortune, Gavelek, & Au, 2014). The relationship between socialization and learning underpins a sociocultural theory of cognitive development (Vygotsky, 1978). Sociocultural theorists view learning as a process rather than a product and maintain that learning is impacted by the learners' experiences and interactions with peers and teachers (Gavelek & Raphael, 1996). Sociocultural theory framed the creation and implementation of a situated apprenticeship professional development (SAPD) program, designed to immerse participants in an active learning experiences situated in their professional context. A total of nine high-school mathematics teachers completed the year-long professional development program that took place from September of 2018 to May of 2019. The nine mathematics teachers participated in the SAPD program as individuals but were organized into subject content teams during SAPD face-to-face sessions.

Professional development programs that focus on educator growth and development result in the implementation of improved instructional practices (Calvert, 2016; Jensen et al., 2016; Learning Forward, 2011). While the main purpose of this research was to increase the use of reform-minded instruction among study participants, the study's treatment theory recognized the impact that teacher knowledge and teacher beliefs have on the implementation of learned practices (Avalos, 2011; Clarke & Hollingsworth, 2002; Guskey, 2002). In addition, the ability of professional development programs to result in change in teachers' practice depends on the quality of the program and the fidelity to which the intervention was implemented (Linnan & Steckler, 2002).

The purpose of this research was to examine high school mathematics teacher' knowledge, beliefs, and implementation of reform-minded instruction throughout the duration of the SAPD program, as well as examining the degree to which the SAPD program was implemented as designed. In this section, I will provide an analysis of the following three outcome research questions:

RQ 1: How did high school mathematics teachers' use of reform-minded instructional practices change as a result of the situated apprenticeship professional development program?

RQ 2: How did high school mathematics teachers' pedagogical beliefs change as a results of the situated apprenticeship professional development focused on reform-minded instruction?

RQ 3: How did high school mathematics teachers' knowledge of reform-minded instructional practices change as a result of the situated apprenticeship professional development program?

as well as an analysis of the following three process evaluation research questions:

RQ 4: What aspects of the situated apprenticeship professional development program do high school mathematics teachers describe as most impactful in their transformation toward reform-minded instruction?

RQ 5: What aspects of the situated apprenticeship professional development program do high school mathematics teachers describe as least impactful in their transformation toward reform-minded instruction?

RQ 6: To what extent was the situated apprenticeship professional development program implemented as designed?

Outcome Evaluation

Fundamentally, learning involves change (Alexander, Schallert, & Reynolds, 2009). Professional development theorists purport that educator learning stems from changes in knowledge and beliefs (Darling-Hammond et al., 2017; Mezirow, 1978) and that effective professional development leads to effective teaching (American Educational Research Association, 2005; Darling-Hammond et al., 2017; Learning Forward, 2011). Researchers recognize the role that teacher content knowledge, pedagogical content knowledge, and beliefs play in mediating instructional change (Desimone, 2009; Guskey, 2002; Roehrig & Kruse, 2005) and promote a sociocultural approach to professional development to as a means of educator growth (Raphael et al., 2014).

A convergent mixed methods approach (QUAN + QUAL) was used to study program outcomes; changes in high school mathematics teachers' knowledge of reform-minded instruction, changes in high school mathematics teachers' beliefs regarding reform-minded instruction, and changes in high school mathematics teachers' implementation of reform-minded instruction. Quantitative and qualitative data informing these outcomes were collected concurrently and merged for analysis (Creswell & Plano Clark, 2011). Framed by the understanding that educator growth is continuous (Learning Forward, 2011) and gradual (Guskey, 2002), and given the relationship between professional learning outcomes and program duration (Darling-Hammond et al., 2017), findings are discussed at three points throughout the SAPD program: early SAPD program (sessions 1 through 3), mid SAPD program (sessions 4 through 7), and late SAPD program (session 8 through 11).

Knowledge of Reform-Minded Instructional Practices (RQ 1)

In this section, I investigate how participants' knowledge of reform-minded instructional practices changed throughout the duration of the SAPD program (RQ1). Quantitative data include participant scores on researcher-created, multiple choice SAPD session exit tickets. Each SAPD session exit ticket contained five multiple choice items designed to assess participants' understanding of specific aspects of reform-minded instruction. The five-item assessments included one item for each of the five subscales of the RTOP: lesson design and implementation, propositional pedagogical knowledge, procedural pedagogical knowledge, communicative interactions, and student-teacher relationships (Sawada et al., 2000). Sources of qualitative data include SAPD session recordings, implementation feedback forms, post-intervention semi-structured interviews, and SAPD session activity artifacts. SAPD session recordings were deidentified and transcribed verbatim. The analysis of SAPD session transcripts involved distinguishing between individual participant perspectives and summaries of small group discussions. Data were attributed to individual participants (pseudonyms) when possible.

To determine whether participants' knowledge of reform-minded instructional practices changed throughout the SAPD program, overall and subscale scores across SAPD session exit tickets were compared. Scores on exit tickets ranged from 0 (0% of items correct) to 5 (100% of items correct). Due to inconsistencies in exit ticket completion rates among participants, quantitative analysis is limited to the use of descriptive statistics. Completion rates of knowledge survey by participants will be discussed further in the process evaluation section of this paper. Table 5.1 outlines the mean scores and standard deviations for SAPD session exit tickets.

Table 5. 1

SAPD Session Exit Ticket Descriptive Data: Knowledge of Reform-Minded Instructional Practices

	Exit Ticket 1 $n = 9$		Exit Ticket 2 $n = 8$		Exit Ticket 3 $n = 9$		Exit Ticket 4 $n = 6$		Exit Ticket 5 $n = 8$	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Overall	4.11	1.05	3.25	1.49	3.22	.97	4.50	.84	4.25	.71

Data illustrate little fluctuation in participant mean score across the five exit tickets. Participant mean scores from the initial exit ticket ($M = 4.11$; $n = 9$) to the final exit ticket ($M = 4.25$; $n = 8$) illustrate consistency and accuracy in participants' ability to identify characteristics of reform-minded mathematics instruction. Data also reflect a slight decrease in standard deviation from the initial (*SD* = 1.05) to the final (*SD* = .71) exit ticket, indicating that participants' responses to exit ticket items converged.

Participant knowledge of reform-minded instruction by RTOP subscale was also analyzed. Inconsistencies in exit ticket completion rates prevented an analysis of subscale items by participant. Rather, an analysis of total survey items by subscale is presented. Table 5.2 presents the overall number of correct and incorrect responses for all participants across exit ticket subscale items. The data reflect a total of 40 responses for each subscale out of a possible 45, for a completion rate of 88.9% (*N* = 9). This information is helpful in determining strengths and weakness in participants' understanding of the characteristics of reform minded instruction.

Table 5. 2

Exit Ticket Subscale Descriptive Data: Group Percent Correct by Subscale (n =40)

RTOP Subscale	Total Correct Responses	Total Incorrect Responses	Correct Response Rate
Lesson Design and Implementation	36	4	90.0%
Propositional Pedagogical Knowledge	24	16	60.0%
Procedural Pedagogical Knowledge	34	6	85.0%
Communicative Interaction	33	7	82.5%
Student-Teacher Relationship	31	9	77.5%

As a group, participant knowledge for the subscale *lesson design and implementation* was the highest. Participants correctly answered 36 out of 40 items for this subscale indicating that participants understood that reform-minded instruction accesses student prior knowledge, encourages exploration and alternative explanations, and engages students in collaborative learning (Sawada et al., 2002). Overall, participants scored the lowest on *propositional pedagogical knowledge* subscale items (60% correct response rate). This subscale includes identifying that reform minded lessons focus on conceptual understanding, employ elements of abstraction, and connect with other disciplines (Sawada et al., 2000). Common errors on propositional pedagogical knowledge subscale items include the misidentification of guided practice as a reform-minded instructional strategy for teaching abstract concepts and identifying skill development as a primary focus rather than concept development. As a cohort, participants also scored lower on *student-teacher relationship* subscale items (77.5% correct response rate) compared with other subscales. Commonly, participants identified patience as a quality of reform-minded instructors, but misinterpreted the role of instructors, identifying instructors as managers rather than listeners.

A benefit of mixed-methods research is the deeper understanding that is derived from the mixing of quantitative and qualitative data (Creswell & Plano Clark, 2011). Illustratively, while quantitative data suggest consistency in participants' knowledge of reform-minded instruction throughout the duration of the SAPD program, qualitative data provide a complementary perspective. Additionally, qualitative data provide an explanation for group performance across subscale items. Sources of the qualitative data include SAPD session recordings, implementation feedback forms, post-intervention semi-structured interviews, and SAPD session activity artifacts. Using a directed approach to content analysis (Hsieh & Shannon, 2005), participants' responses were categorized using RTOP subscales as themes. SAPD program foci served as a priori codes and included *prior knowledge, conceptual understanding, student* *exploration, student discourse,* and *teacher as facilitator.* Several emergent codes arose during the data analysis process. The first set of emergent codes; *collaboration*, *connecting, hypothesize, student reflection,* and *student led,* are characteristic of reform-minded instruction (Sawada et al., 2002) but were not foci of the SAPD program. A second set of emergent codes included *planning time, implementation time, and general teaching efficacy.* The emergence of these codes may indicate that participants incorrectly view reform-minded instruction as a supplement to daily mathematics instruction and as too rigorous for some students. Mathematics teachers commonly and mistakenly reserve rigor for high-ability mathematics students (Powell et al., 2013; Pugach & Warger, 2001) which may explain participants' views regarding general teaching efficacy. An external reviewer confirmed a priori and emergent codes and suggested an additional emergent code of *self-reflection.* The external reviewer noted that in semi-structured interviews, some participants identified knowledge of reform-minded instruction as an area of personal growth. A list of codes and themes are provided in Table 5.3.

Table 5. 3

A Priori and Emergent Codes Aligned with RTOP Subscales: Qualitative Indicators of Knowledge of Reform-Minded Instructional Practices

RTOP Subscale	A Priori Codes	Emergent Codes
Lesson Design and Implementation	Prior knowledge Student exploration	Collaboration Planning time
Propositional Pedagogical Knowledge	Conceptual understanding Connecting	Implementation time General teaching efficacy
Procedural Pedagogical Knowledge		Multiple representations Clarification
Communicative Interactions	Student discourse	
Student-Teacher Relationships	Teacher as facilitator	Student led
		Self-reflective

Early SAPD program. SAPD session recordings and SAPD activity artifacts provide evidence of participants' ability to identify characteristics of reform-minded mathematics instruction. In session two, and in subsequent sessions thereafter, participants experienced, discussed, and planned reform-minded lessons. As early as the first SAPD session, participants were able to identify characteristics of reform-minded instruction corresponding to the subscales of lesson design and implementation and student-teacher relationships. The subsequent analysis of early SAPD program findings provides an explanation for discrepant quantitative findings.

During the first and second SAPD sessions, participants engaged in activities comparing traditional instruction with reform-minded instruction. Participants recognized that encouraging students to explore mathematics content prior to formal instruction (Swada et al., 2000) was an essential aspect of designing reform-minded lessons. Audible comments from early SAPD session TBT discussions include “investigating,” “discovery,” “exploration,” and “problem-solve.” Although students are not expected to discover mathematics concepts independently in reform-minded lessons, participants seemed to equate discovery with exploration. Illustratively, participant 6 stated “in reform-minded lessons, students problem-solve and discover” and participant 1 noted that “in reform-minded [lessons] there is more discovery going on rather than just lecture.” Recognition of these characteristics in the early stage of the SAPD program align with quantitative findings that suggest that participants were knowledgeable of lesson design and implementation subscale items. Participant knowledge of the elements that contribute to designing and implementing reform-minded mathematics lessons help to explain the high mean score on the initial exit ticket ($M = 4.11$; $SD = 1.05$; $N = 9$).

Participant responses throughout the early stages of the SAPD program indicate an awareness that the role of the teacher in reform-minded instruction is that of facilitator. The recognition of instructors as facilitators reflects an understanding of how student-teacher relationships in reform-minded classrooms differ from the relationships established in traditional classrooms (Sawada et al., 2002). Participants noted that reform-minded instructors, “walk[ed] around” and “were more facilitative.” When reporting key points of a small group discussion, participant 8 provided the following description of the role of a reform-minded instructor: “the teacher is in the background guiding the questions.” SAPD session activity artifacts provide further evidence of participants’ knowledge of the role of teachers in reform-minded instruction. When asked to collaborate on a list of concrete steps that could be taken to move away from traditional instruction and toward reform-minded instruction, one group of participants listed “let students lead.” Quantitative findings suggest that participants were less accurate in responses to student-teacher relationship subscale items (77.5%) compared with other subscale items. However, early SAPD exit ticket items focused solely on the role of instructors in reform-minded classrooms and early qualitative data indicate that participants understand the role of the teacher in reform-minded instruction.

Mid SAPD program. Sessions four through seven involved participants in designing and implementing reform-minded instructional strategies. SAPD session activities and classroom modeling sessions were designed to increase participants' knowledge of reform-minded instructional practices, knowledge categorized as informational knowledge (Rohlwing & Spelman, 2014). For adult learners, learning is a social process (Raphael et al., 2014) requiring self-reflection (Mezirow, 1997) and productive struggle. During this stage of the SAPD program, evidence suggests that change was occurring. Mean scores on exit tickets 2 and 3 (M = 3.25 and M = 3.22 respectively) suggest a decrease in participant knowledge of reform-minded instruction. Qualitative data provide an explanation for the drop in mean score as well as providing an explanation for the differences in correct response rates across subscale items.

Evidence of participant knowledge of reform-minded instruction during sessions four through seven of the SAPD program was collected from SAPD session recordings, SAPD session artifacts, and implementation feedback forms. Qualitative data predominantly aligned with the RTOP subscales of lesson design and implementation, propositional pedagogical knowledge, procedural pedagogical knowledge, and communicative interactions. Propositional pedagogical knowledge considers the implementation of lessons with a strong conceptual focus and an understanding of how mathematical concepts are connected to each other and to other disciplines (Ebert-May et al., 2011; Sawada et al., 2000). Procedural pedagogical knowledge focuses on the behaviors of the students in reform-minded classrooms (Ebert-May et al., 2011) and addresses the activities in which students engage (Sawada et al., 2002). Similarly, reform-minded communicative interactions promote student discourse as a source of instruction and pacing (Sawada et al., 2002). Together these subscales address characteristics of implementing reform-minded lessons.

Although participants consistently responded correctly to questions pertaining to lesson design and implementation (90%), qualitative data suggest that participants held varying perceptions regarding lesson design elements of reform-minded instruction. Participants' reflections on a video clip of a master teaching a student karate provided evidence of a firm understanding of the differences between traditional and reform-minded instruction. Participant 2 described the style of instruction as reform-minded and noted that "had it been traditional, [the instructor] would have taught [the student] directly" and that the student "does not even know he learned karate." In their reflections, participants continued to describe reform-minded instruction as investigative and as one participant noted, reform-minded instruction involves gaining "insight into what [the students] are really thinking" (Participant 3). Conversely, implementation feedback forms provided an indication that perhaps participants lack a depth of knowledge regarding the lesson design and implementation of reform-minded instruction. For example, responses on implementation feedback forms indicated that participants view their students' partner work and group work as reform-minded, even when the work focused only on skills rather than concepts or a mixture of concepts and skills. In another example, participant 4 mistakenly categorized working in groups to solve surface area problems as reform-minded even though students were given all the dimensions of the solids making it a very routine task. Additionally, participant 5 noted favorably that an instant grading feature on a computerized program provided "a good way of identifying specific mistakes students may be making." This participant seemed to have categorized all types of feedback as reform-minded. Unlike the detailed, individual student feedback provided by reform-minded instructors (Ebert-May et al., 2011), this computerized program provided limited feedback, identifying student responses as *correct* or *incorrect.*

Indicators that reform-minded instruction promotes a conceptual understanding of mathematics and involves making connections between concepts, elements of propositional pedagogical knowledge (Sawada et al., 2002), began to emerge throughout SAPD sessions 4 through 7. In one SAPD session, participants were given the opportunity to reflect on reform-minded lessons that had been recently modeled by the program facilitator in participants' respective classrooms. After observing the delivery of a reform-minded lesson on factoring quadratic functions utilizing a Desmos classroom activity (Desmos, 2019), participant 9 noted that "everything ties together, factoring and finding the *x*-intercepts, everything fits together." In another SAPD session, participants engaged as students in a 3 Act Math task. When reflecting on the task, participant 4 remarked that the task was reform-minded in that it provided a focus on "conceptual understanding" and participant 6 noted that the 3 Act Math task "gave [the students] something to connect [the information] to." Exit ticket data indicate that participants incorrectly responded to propositional pedagogical knowledge items 40% of the time. This quantitative indicator of participants' propositional pedagogical knowledge was limited to the multiple choices presented and as qualitative data illustrate, participants demonstrated an understanding that reform-minded instruction is rooted in a conceptual focus.

NCTM promotes the development of procedural fluency through conceptual understanding (2014). An explanation for participants' errors on propositional pedagogical knowledge subscale items (40% incorrect response rate) is that participants tended to view mathematics lessons that focused on concepts as an addition to the curriculum rather than an instructional design. Participants acknowledged that reform minded instruction involved "investigations into concepts" however, participants consistently misidentified conceptual lessons as those that were the "creative stuff" (Participant 5) that could be implemented "if you did not have to get ready for the [end-of-course] test" (Participant 5). Participants worried that "non-calculation activities take away from calculation practice" (Participant 7). The following exchange illustrates that some participants perceive lessons with a procedural focus as more important than lessons with a conceptual focus, indicating a misinterpretation regarding propositional pedagogical knowledge:

Participant 2: You have to have some time of practice. [Students] can explore parameters, but they have to take a look at some examples.

Facilitator: How else is time a challenge?

Participant 10: If we didn't have a curriculum, [we] could spend all the time [we] want on everything. We have to get through things and make sure [the students] know what is on the [end-of-course] test.

This exchange illustrates that participants continue to perceive procedural skills as more important that a conceptual understanding of mathematics, a perception indicative of a traditional approach to mathematics instruction (Boaler & Staples, 2008).

Reform-minded instructional strategies include encouraging learners to clarify thoughts and explain reasoning (Sawada et al., 2002). Although these elements were outside of the SAPD program focus, SAPD program resources contained information on these tenets of reform-minded instruction. Through the analysis of SAPD session recordings and lesson implementation forms, the code of clarification emerged, providing evidence that participants were developing a deeper understanding of reform-minded instruction. As described by participants, asking students to clarify solutions "gives us insight into what [the students] are really thinking" (Participant 2) and "gives [students] a chance to explain their thoughts" (Participant 3). After implementing a reform-minded lesson, participants noted that "[students] are not used to having to explain their reasoning" (Participant 5) and that "explain[ing] why polynomials are named as they are" (Participant 4) was an effective reform-minded technique.

Sociocultural theory promotes understanding through communication (Cobb, 1994; Vgotsky, 1978). Likewise, student discourse forms the foundation of reform-minded instruction (Kozulin, 2004; Sawada et al., 2002). Based on participants' correct response rate on communicative interaction subscale item (82.5%), participants generally recognize the importance of student discourse to reform-minded instruction. Qualitative data confirm quantitative findings. On lesson implementation feedback forms, participants identified "discussion between students" (Participant 1) as a reform-minded strategy and stated that reform-minded lessons provide students an opportunity to "explain to each other why polynomials are named as they are" (Participant 4). Noting the importance of student discourse, one participant stated that "students lack fluency with using proper vocabulary in partner discussions" (Participant 7), but in a subsequent SAPD session described student discourse as, "[students] serving as the instructor to other student[s] (Participant 7). Student discourse represents only one aspect of the communicative interaction subscale (Sawada et al., 2002). The fact that participants did not refer to promoting divergent thinking or multi-modal forms of communication (Sawada et al., 2002) when describing lessons confirms that participants' knowledge was in the developmental stage at this midway point in the SAPD program.

Late SAPD program. Qualitative data, collected from SAPD session recordings, SAPD activity artifacts, and post-intervention semi-structured interviews, help to explain participant knowledge in greater detail than the quantitative data alone. Mean scores on exit three and four ($M = 4.5$; $SD = .84$; $n = 6$ and $M = 4.25$; $SD = .71$; $n = 8$ respectively) were a marked increase from mean scores on exit tickets two and three ($M = 3.25$; $SD = 1.46$; $n = 8$ and $M = 3.22$; $SD = .97$; $n = 9$ respectively). Qualitative data from SAPD recordings and artifacts for sessions 8 through 11 help to provide a deeper understanding of how participants' knowledge continued developed during this stage in the SAPD program. Consistent with qualitative findings of early SAPD sessions, data indicate that participants can identify general characteristics of reform-minded instruction. Additionally, data illustrate that teachers' challenges in understanding propositional pedagogical knowledge persist. An additional theme of self-reflection emerged during the analysis of participant responses during semi-structured interviews.

Qualitative data provide confirmation of participants' understanding that reform-minded instruction involves focusing on concepts rather than skills. Participants accurately identified teaching with a conceptual focus as a factor differentiating reform-minded from traditional instruction. Participants recognized that reform-minded instruction "increases [students] understanding of math concepts" (Participant 1) and reform-minded lessons require that students "have a true understanding of the concept and not just memorize" (Participant 3).

According to exit ticket subscale data, participants correctly answered propositional pedagogical knowledge items only 60% of the time. Qualitative data from late stage SAPD sessions (session 8 through 11) offer an explanation for this low accuracy rate. Data from SAPD session recordings suggest that participants continue to equate reform-minded instruction with any form of group work. For example, participant 5 incorrectly indicated that a recent lesson requiring students to work in groups to complete procedural tasks qualified as reform-minded. Referring to this lesson, participant 5 stated: "they basically know who is smarter and they would just let the one person solve [the problem] and copy the answer." Furthermore, participants continue to have the misunderstanding that reform-minded instruction fails to teach skills and procedures, rather than recognizing that procedural skills follow from lessons with a conceptual focus (NCTM, 2014).

> My biggest concern [or] worry is that by fully implementing reform-minded activities, I could lose valuable instructional time and risk my students not developing [or] retaining the necessary skills. (Participant 7)

This participant's reflection illustrates that participants perceive reform-minded instruction as detracting from procedural fluency.

SAPD session recordings and artifacts provided evidence of participants' procedural pedagogical knowledge. In a later SAPD session activity, participants were asked to brainstorm ways to transform traditional questioning techniques to reform-minded techniques. Participant-created reform-minded questions included "is there another way to solve this problem" (Participant 1), "can you draw a picture" (Participant 6), and "what strategy did you use to solve this problem" (Participant 2). In a second activity, participants were asked to identify reform-minded elements of an instructional strategy suggested in a What Works Clearinghouse practice guide (Star et al., 2015). Participants specified the key words of "reasoning," "multiple representations," and "multiple solutions," as reform-minded elements. Knowledge of multiple representations as a reform-minded element is also reflected in the following discourse:

> Facilitator: "What makes this lesson reform-minded?"
>
> Participant 3: "I had the students show the class another way to solve a kite problem."

Facilitator: “What is the benefit of seeing multiple solutions?”

Participant 3: “I might rely on the kite being 360 degrees. That is what sticks with me. But [my co-teacher and I] hammered [the students] with triangles. It was an elaborate way to solve the kite, but the kids understood it. They preferred that method.

Participant 3 not only encouraged students to explore variety of strategies when solving kite problems, but recognized that in doing so, students could conceptualize the mathematics content in a personal manner.

An examination of mean scores on exit tickets from the first exit ticket (M = 4.11; SD = 1.05; n = 9) to the final exit ticket (M = 4.25; SD = .71; n = 8) suggest that participant knowledge remained relatively consistent through the SAPD program. Contrary findings were reflected in participant responses during semi-structured interviews. The theme of self-reflection emerged from participant responses as participants identified knowledge of reform-minded instruction as an area of personal growth. Participant 4 explained, “I did not have a firm understanding of reform-minded instruction before [the facilitator] talked about it” and noted “a lot of similarities [between] project and problem-based learning and reform-minded instruction.” Likewise, participant 8 found that “everything was new to me, I really did not have any prior knowledge.” Participant 5 indicated that the information of reform-minded instruction taught “how to engage students in different ways” compared to direct instruction and participant 7 desires “a follow-up program based on new research findings in order to keep the growth moving forward.” The difference between quantitative measures of participant knowledge and participants’ perceptions of growth can be attributed to the fact that quantitative findings represent whole-group, mean changes in knowledge which might conceal individual growth, whereas responses during semi-structured interviews represent individual changes in knowledge.

As a measure of participants' knowledge of the characteristics of reform-minded instruction, qualitative data prevent a more comprehensive understanding compared with quantitative data. Highlights from participant responses, representative of knowledge at the three key points in the SAPD program is displayed in Table 5.4.

Table 5. 4.

Summary of Participants' Description of Reform-Minded Instruction at Three Points in SAPD Program

Time	Participants Description of Reform-Minded Instruction	RTOP Subscale
Early SAPD Program	"In reform-minded [lessons] there is more discovery going on rather than just lecture." (Participant 1) "In reform-minded lessons, students problem-solve and discover" (Participant 6)	Lesson Design and Implementation
	"The teacher is in the background guiding the questions." (Participant 8) "Let students lead" (Small group)	Student-Teacher Relationships
Mid SAPD Program	"Had it been traditional, [the instructor] would have taught [the student] directly." (Participant 2)	Lesson Design and Implementation
	"If we didn't have a curriculum, [we] could spend all the time [we] want on everything. We have to get through things and make sure [the students] know what is on the [end-of-course] test." (Participant 10)	Propositional Pedagogical Knowledge
	"Gives us insight into what [the students] are really thinking" (Participant 2) "[Students] are not used to having to explain their reasoning" (Participant 5)	Procedural Pedagogical Knowledge

Table 5.4 (continued)

	"[Students] serving as the instructor to other student[s] (Participant 7).	Student-Teacher Relationships
Late SAPD Program	"I had the students show the class another way to solve a kite problem." (Participant 3) "Is there another way to solve this problem" (Participant 1)	Procedural Pedagogical Knowledge
	"They basically know who is smarter and they would just let the one person solve [the problem] and copy the answer." (Participant 5) My biggest concern [or] worry is that by fully implementing reform-minded activities, I could lose valuable instructional time and risk my students not developing [or] retaining the necessary skills. (Participant 7) Reform-minded instruction "increases [students] understanding of math concepts" (Participant 1) and requires that students "have a true understanding of the concept and not just memorize" (Participant 3).	Propositional Pedagogical Knowledge
	"Discussion between students" (Participant 1)	Communicative-Interactions

Mathematics Teacher Pedagogical Beliefs (RQ2)

In this section, I investigate how participants' pedagogical beliefs regarding reform-minded mathematics instruction and classroom discourse changed throughout the duration of the SAPD program. Research on educator professional development indicates that teacher beliefs influence instructional practices and that in turn changes in instructional practices involve changes in pedagogical beliefs (Clarke & Hollingsworth, 2002; Desimone, 2009; Guskey, 2002). Therefore, an investigation into the extent to which professional development, such as the SAPD program, influences pedagogical beliefs is warranted.

To investigate potential changes in pedagogical beliefs during the course of the SAPD program, pre- and post-intervention data were collected using the MTPDBI (Lischka & Garner, 2016). Authors of the MTPBDI (Lischka & Garner, 2016) provide two reliability scores. The item separation reliability score of .96 indicates the consistency to which the survey separated responses on a continuum from teacher-centered to learner-centered pedagogical beliefs. Authors attribute a lower personal reliability score (.71) to the sample size ($N = 187$) used when developing the instrument (Lischka & Garner, 2016).

The MTPDBI (Lischka & Garner, 2016) measures pedagogical beliefs on a continuum from teacher-centered to learner-centered. Overall scores on the instrument range from a minimum of 20, indicating teacher-centered beliefs, to a maximum of 80, indicating learner-centered beliefs. Instrument items are divided into four categories of beliefs; teaching, learning, discourse, and the nature of mathematics. The instrument includes 5 items for each category with category scores ranging from 5 to 20. Participants' pre- and post-intervention MTPDBI scores are provided in Table 5.5.

Table 5. 5

Participants Pre-Intervention and Post-Intervention MTPDBI Scores: Measure of Beliefs Regarding Reform-Minded Instruction

	Pre-Intervention MTPDBI Score	Post-Intervention MTPDBI Score
Participant 1	61	66
Participant 3	58	70
Participant 4	68	66
Participant 5	54	63
Participant 6	45	61
Participant 7	55	67
Participant 8	56	59
Participant 9	44	58

A Wilcoxon Signed Ranks test comparing participants' pre-intervention and post-intervention scores was conducted. The non-parametric Wilcoxon Signed Ranks test was chosen because the distribution of differences in participants MTPDBI scores indicated moderate skewness (M_{diff} = 8.63; SD_{diff} = 6.14; skewness = -.652; kurtosis = -.564; n = 8). The change in participants' scores from pre-intervention to post-intervention was found to be significant (p = .017), indicating that participant beliefs changed throughout the SAPD program. The mean difference between pre-intervention (M = 55.13; SD = 7.9; n = 8) and post-intervention (M = 63.75; SD = 4.2; n = 8) instrument scores was 8.63. A significant increase in MTPDBI score illustrates a move from teacher-centered views toward learner-centered views (Lischka & Garner, 2016).

Wilcoxon Signed Ranks tests were also conducted for each of the 4 subscales of the MTPBDI. While mean scores for all MTPDBI subscales increased from pre- to post-intervention, changes in mean scores for the teaching (M_{diff} = 2.88; SD_{diff} = .25; n = 8), learning (M_{diff} = *1.62*; SD_{diff} = -.67; n = 8), discourse (M_{diff} = 1.87; SD_{diff} = -.4; n =8), and the nature of mathematics (M_{diff} = 1.00; SD_{diff} = .65; n = 8) subscales were not significant (M_{diff} = p = .051; p = .127; p = .121; p = .244 respectively). Discrepancies between resulting p-values for composite score versus subscale scores can be attributed to the smaller range of subscale scores (5 – 20) compared with the range for composite score (20-80). Although subscale data provides insight into participants' beliefs regarding teaching, learning, discourse, and the nature of mathematics, subscales were not intended to measure learner- or teacher-centeredness independently (Lischka & Garner, 2016). 5.6 displays descriptive composite and subscale data for pre- and post-intervention administrations as well as composite and subscale Wilcoxon Signed Ranks test results.

Table 5. 6

Summary of Composite and Subscale Descriptive and Inferential Statistics on the MTPBDI

Mathematics Teaching Pedagogical and Discourse Beliefs Instrument	Pre $n = 8$		Post $n = 8$		Wilcoxon Signed Ranks Test	
	M	SD	M	SD	z	p
Composite Score	55.13	7.9	63.75	4.2	-2.383^b	.017*
Subscales						
Teaching	13.00	3.02	15.88	3.27	-1.951^b	.051
Learning	13.88	3.23	15.50	2.56	-1.527^b	.127
Discourse	14.63	2.33	16.50	1.93	-1.550^b	.121
Nature of Math	13.63	1.85	14.63	2.50	-1.166^b	.244

Professional development programs seeking to alter educator beliefs recognize change as a gradual process (Guskey, 2014). Although changes in participant beliefs regarding teaching, learning, discourse, and the nature of mathematics were insignificant, qualitative data provide insight into the change in beliefs that occurred for individual participants.

Sources of qualitative data assessing participants' pedagogical beliefs include SAPD session recordings, post-intervention semi-structured interviews, implementation feedback forms, and SAPD session artifacts. I employed a directed approach of analysis (Hsieh & Shannon, 2005) using the MTPDBI construct map (Lischka & Garner, 2016) (see Appendix I) to formulate a priori codes and themes. Participant comments and responses were coded and categorized into the themes of *Learner-Centered, Approaching, Emerging,* and *Teacher-Centered* to align with MTPDBI categories. Several emergent codes arose during the analysis process. Participants referenced *planning time, instruction time, instructional change,* and *general self-efficacy,* when discussing beliefs regarding reform-minded mathematics pedagogy. These emergent codes signified constructs that participants perceived as obstacles to reform-minded instruction and were organized under the theme of *Immunities to Change* (Helsing, Howell, Kegan, & Lahey, 2008). Table 5.7 provides a summary of a priori codes, arranged by MTPDMI (Lischka & Garner, 2016) subscale and theme, along with emergent codes.

Table 5. 7

A Priori and Emergent Codes from Qualitative Sources of Data Aligned with RTOP Subscales

Theme	Code	
	MTPDMI Subscale	
Learner-Centered	Teaching	Teacher as facilitator
	Learning	Knowledge through exploration
	Discourse	Student directed discourse
	Nature of Math	Math as creation
Approaching	Teaching	Teacher as explainer
	Learning	Knowledge as problem solving
	Discourse	Teacher directed discourse
	Nature of Math	Math as discovery
Emerging	Teaching	Teacher as instructor
	Learning	Knowledge as procedures
	Discourse	Question/response discourse
	Nature of Math	Math as connected facts
Teacher-Centered	Teaching	Teacher as informer
	Learning	Knowledge as memorization
	Discourse	Lecture
	Nature of Math	Math as facts and skills
Immunities to Change		*Planning time*
		Instructional time
		Instructional Change
		General teaching efficacy

Note. Emergent codes appear in italics

Early SAPD program. The initial three SAPD sessions endeavored to explore participants' current pedagogical beliefs and instructional practices and to introduce participants to reform-minded pedagogy. Whole-group discussions following SAPD activities and session artifacts provide a rich source of qualitative data on participants' pedagogical beliefs during the early stages of the SAPD program. Research suggests that educators' professed beliefs regarding reform-minded instruction often contradict portrayed beliefs (Cross, 2009). While quantitative data suggest that participants' beliefs were more learner-centered than teacher-centered (M = 55.13; SD = 7.9; n = 8), qualitative data indicate greater variation in participants' beliefs.

At the onset of the SAPD program, participants were asked to collaborate on writing a list of barriers to student success in high-school mathematics. This program artifact serves as an indicator that participants generally hold teacher-centered pedagogical beliefs. Participants wrote that students "need more drill," "lack prerequisite skills," "don't practice enough," "lack [the] basics," fail to "follow directions," "forget essential skills," have problems with "retention." Such comments seem to indicate that participants view mathematics as a collection of facts and skills and that learning mathematics involves rote memorization of procedures, views consistent with teacher-centered pedagogical beliefs (Lischka & Garner, 2016). Conversely, pre-intervention MTPDMI scores indicate that participants hold emerging and approaching (Lischka & Garner, 2016) beliefs. Participants' pre-interventions MTPDMI scores ranged from 44 to 68 (M = 55.13; SD = 7.9; n = 8). Although MTPDMI scores do not strictly characterize teachers as having teacher-centered or learner-centered beliefs, higher scores are associated with movement toward learner-centered beliefs (Lischka & Garner, 2016).

Individual participant beliefs were reflected in SAPD session recordings and data support the prevalence of teacher-centered beliefs for a large percentage of participants (37.5%, n =8). In the initial SAPD session, participants were given the opportunity to characterized their teaching style and compare it to descriptions of instructors who had a range of RTOP scores. For this activity, RTOP scores from 0 to 30 indicated teacher-centered pedagogy, scores from 31 to 45 indicated teacher-guided pedagogy, scores from 46 to 60 indicated student-influenced pedagogy, and scores above 60 indicated student-centered pedagogy (Classroom Teaching Style Survey, 2016). Participants were asked to consider Dr. Frodo, a teacher-centered instructor described as authoritative and favoring lecture. When asked to reflect on Dr. Frodo's teaching style, fifty percent of participants (N = 9) indicated that they would want to be a student of Dr.

Frodo. Participant 9 recalled having a mathematics teacher like Dr. Frodo and commented, “I loved my math teacher.” Furthermore, participant 9 believed that “kids would be more successful in math if they [were] drilled more in basic facts,” a belief consistent with traditional mathematics instruction (Boaler & Staples, 2008). These remarks may help to explain the low (44) pre-intervention MTPDBI score for participant 9. Participant 2 concurs with participant 9 and believes that the high school mathematics program should “focus more on foundational [skills]. Participant 2 adds that “all of us learned algebra and geometry that was [largely] traditional and we all turned out ok.” The belief that mathematics instruction should focus on the memorization of facts and skills is indicative of teacher-centered beliefs (Lischka & Garner, 2016) and contradicts research findings supporting reform-minded instruction to improve student achievement in mathematics (Krupa & Confrey, 2017).

Early SAPD session recordings and artifacts include explicit mention of emerging and approaching beliefs by some participants, which better aligned with quantitative data. When reflecting on personal aspirations for students, participant 7 wished that students “[had] the language in order to communicate about math.” When asked to elaborate on how to engage students in discourse participant 7 suggested that teachers “come up with different ways to develop questions.” In another SAPD session, participants were asked to comment on the diagram in Figure 5.1 (Bryk, 2017).

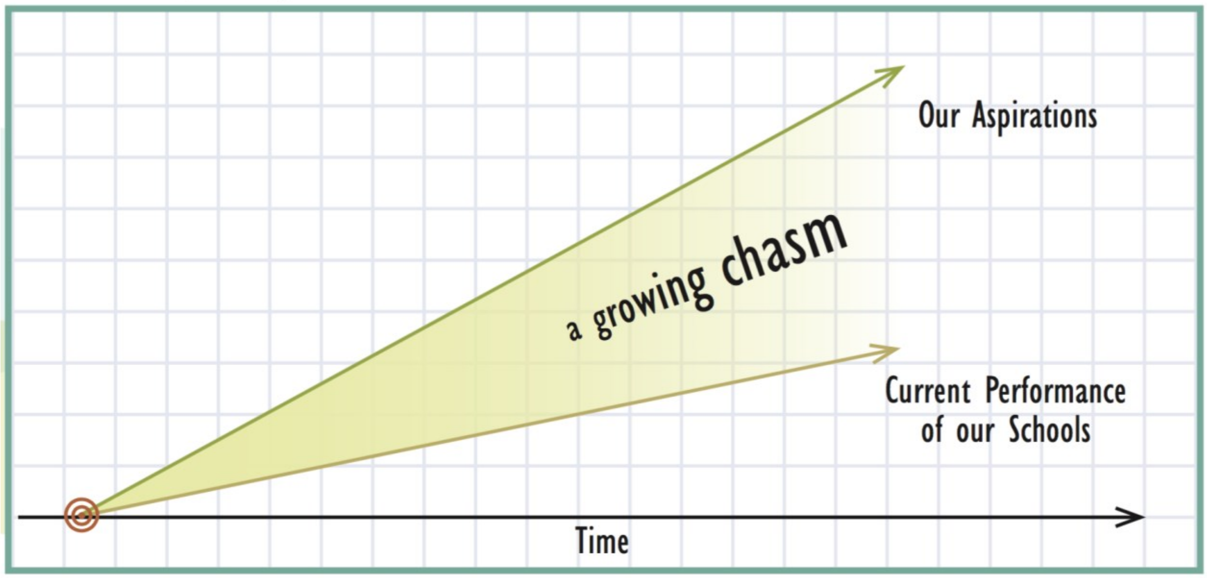

Figure 5. 1. Image serving as SAPD session discussion prompt. Intended to illustrate the gap between current levels of instruction and desired levels of performance (Bryk, 2017).

While many comments reflected the teacher-centered views, participants 1 and 3 expressed that students need to become problem-solvers rather than knowledge accumulators.

> I don't think (end-of-course exam) questions are homing in on random skills. The test is trying to get them to problem solve. I tell my kids you probably aren't going to use half of what we learn in Algebra 2, but you are going to need to know how to problem solve (Participant 3).

Although participant 3 recognizes that mathematics instruction involves more than skill development, participant 3 alludes to teaching material for which students have little application outside of the classroom. Participant 1 also places importance on problem solving. In response to a peer who expressed teacher-centered beliefs, participant 1 stated:

> I think that to prepare kids for where they are going to be today, we need to focus more on [concepts] than specific [procedures]. Because the fact is when they get out of here, if

> they need to know how to factor a quadratic, they can go to a computer and factor it. It needs to be a process of putting kids in problem solving situations.

The expressed beliefs of participant 1 at this juncture in the SAPD program concur with this participant's pre-intervention MTPDBI score of 68 (M=55.13; n = 8). Higher MTPDBI scores reflect beliefs that are more learner-centered than teacher-centered. Conversely, the reflection provided by participant 3, quoted above, is inconsistent with this participant's pre-intervention score of 58 (M=55; n = 8). The conflict between recognizing the value of problem-solving and focusing on non-applicable skills may provide an explanation for this discrepancy.

Early SAPD session artifacts illustrate that participants believe that reform-minded mathematics instruction requires additional instructional time and is reserved for high-ability students. The codes of instruction time and general teaching efficacy were labeled as immunities to change, given that they represent barriers (Helsing et al., 2008) to adopting reform-minded pedagogical beliefs.

In the initial stages of the SAPD program, participants expressed beliefs that reform-minded instruction is a supplement to instruction rather than an instructional approach, and expressed concerns that implementing reform-minded lessons takes time away from addressing standards. Progressing through course standards remains a concern for teachers of state-tested subjects (Au, 2007), providing a possible explanation for participants' concerns regarding time. For example, participants noted that reform-minded instruction would be possible if "we had more time for teaching" or "if there wasn't state testing." When reflecting on reform-minded instruction, participant 9 stated that "we can't spend a lot of time on [reform-minded lessons] because there is so much we have to teach." Additionally, one participant expressly identified time as a barrier when stating "I did not do any modeling. I did not have time."

Participants also expressed beliefs that student ability impacts the feasibility of employing reform-minded pedagogy. In an early SAPD session activity, participants were asked to discuss how reform-minded instruction aligns with aspirations we hold for our students. Participant 7, an intervention specialist working with students with learning disabilities, remarked "some of the aspirations I have for my students are not necessarily those I share for typical students." Additionally, participant 6, also an intervention specialist, noted that conceptual learning is difficult for "our students." Remarks such as these suggest that these participants may have lower general teaching efficacy for students identified with learning disabilities. Low general teaching efficacy provides an explanation for the pre-intervention MTPDBI scores for participants 6 and 7 (45 and 55 respectively; M=55.13; n = 8).

Mid SAPD program. Qualitative data suggest that participants' pedagogical beliefs began to broaden during the middle portion of the SAPD program. The classroom modeling component of the SAPD program commenced at this time and participants began implementing reform-minded strategies introduced during face-to-face sessions. As Guskey (2002) maintains, implementation successes contribute to changes in pedagogical beliefs. Participants' reflections on implementation feedback forms suggest that participants experienced small successes which helps to explain the movement of beliefs along the continuum from teacher-centered to learner-centered. Qualitative data collected from SAPD session recordings and implementation feedback forms suggest that participant beliefs expanded to include all four themes as described in Table 5.7.

Data consistent with learner-centered beliefs emerged for the first time at this midpoint in the SAPD program. As indicated by high pre- and post-intervention MTPDBI scores (61 and 66 respectively) the pedagogical beliefs of participant 1 align with the themes of approaching and learner-centered. Countering group concerns that reform-minded teaching takes time way from teaching necessary procedures and skills, participant 1 remarked that "the time investment will be worth it." Participant 1 created and implemented a lesson requiring students to design a school building using conic sections. Participant 1 noted that "at first, [students] want specific direction as to what the [school] must look like" and that "it [was] hard for some groups to get past not being told exactly what to do." However, participant 1 saw the lesson as "a great way to have students apply conic sections to something new to them." Additionally, participant 1 explained how reform-minded instruction requires a change in thinking: "With reform-minded lessons, kids can take you in another direction. They take longer than you think. You have to be willing to adapt within the lesson itself." This reflection represents this participant's willingness to let student discourse influence the direction of the instruction.

Consistent with early SAPD program qualitative data and pre-intervention MTPDBI scores in the upper 50's and 60's, some participants hold pedagogical beliefs that fall in the category of approaching. Evidence of approaching beliefs is found in this summary following a small-group discussion on reform-minded implementation successes:

> It's not about showing [the students] different ways, but really trying to help them see connections between everything. That what I focus a lot on. You have to get down to the core concept and see how you can explain that. Establish connections. (Participant 3)

Participants recognize the importance of a conceptual focus in mathematics instruction, but continue to view teachers as explainers rather than facilitators.

An indication of early changes in pedagogical beliefs occurred in responses made by participant 6. Participant 6 received a score of 45 (M= 55.1; n =8) on the pre-intervention MTPDBI instrument. In early SAPD sessions, participant 6 expressed concern that reform-minded instruction would be difficult for students diagnosed with learning disabilities. Conversely, comments made by participant 6 following a reflection on the image in figure 5.2 indicate that participant 6 believes in exploration as a component of mathematics instruction.

Figure 5. 2. Image serving as SAPD session discussion prompt. Intended to use symbolic representations to depict characteristics of reform-minded instruction.

> I was thinking the bottom is how we teach our students when they are young and then we stop. When they get older it is more, 'you are going to listen and write this down.' We need more of [what is depicted at] the bottom. More play. (Participant 6)

Participant 6's self-reported success with implementing the reform-minded strategies introduced in SAPD sessions is a plausible explanation for these early changes in beliefs.

Qualitative data also suggest movement along the learner-centered to teacher-centered continuum for participant 2. Although incomplete responses prevented a quantitative measure of pedagogical beliefs for participant views, early qualitative data suggested that participant 2 held teacher-centered beliefs and preferred traditional over reform-minded instructional strategies. Current qualitative data suggest that participant 2 is challenging earlier expressed beliefs, especially in the subscale of learning.

> I am always torn between mechanics and context. Part of me says you need to know how to divide polynomials but another part says with technology being what it is calculators will be able to do that stuff for [the student]. In terms of error analysis, I like [this strategy] because [the students] can think about the misstep. [The students] become the teacher. It's that idea of how much we should focus on mechanics. (Participant 2)

On an implementation feedback form, participant 2 commented that error analysis lesson allowed students to experience "that ah-ha moment of realizing that the error was not checking for an extraneous solution" which is indicative of reform-minded lesson. After implementing a reform-minded lesson involving solving rational equations, participant 2 reflected, "I like [error analysis] problems as [they] strengthen content knowledge through student [self] grading and teaching."

Given that changes in pedagogical beliefs are generally gradual (Guskey, 2002) it was not surprising to find that some participants continued to express teacher-centered pedagogical beliefs. Qualitative data suggest that participant 5 separates real-world from in-school mathematics and focuses on skills rather than concepts. Likewise, participant 9 seemingly believes in teaching content over context. The teacher-centered beliefs of participants 5 and 9 are reflected in the following conversation:

> Participant 5: You don't have to be a problem solver to get the questions right. If you teach a tested subject, you are judged by [whether or not] your [students] get the questions right.

Participant 10: In Algebra 2, I used to cover 10 chapters and now I get through 6. [The students] do not really got to algebra 2 [material]. They are just not going to do it.

Participant 5: But in the real world you would have to be a problem-solver

After implementing lessons requiring student exploration with their classes, teachers report that students found the lessons "confusing" (Participant 9) and that "students were not used to explaining" (Participant 5). Such comments indicate that reform-minded instruction is a change for students as well as teachers. Given that student outcomes mediate changes in educator beliefs (Cross, 2009; Guskey, 2002), students' frustrations during implemented lessons could explain why some participants continue to demonstrate teacher-centered instructional beliefs.

Instructional time and general teaching efficacy represent continued immunities to change for several participants. Misunderstanding of propositional pedagogical knowledge, as explained in the analysis of RQ1, offer a possible explanation for participants' concerns regarding time. Specifically, participant 2 expresses that time constraints impact the feasibility of implementing reform-minded instruction. "Time wise, are [teachers] able to do this? We only have 45 minutes each class period and 180 days. There is only so much [we] can do with the time you have" (Participant 2).

For other participants, general teaching efficacy, especially toward students who struggle with mathematics, is an area of concern. Participant 3 worries that students "are not used to reform-minded teaching. [These students] do not know what to do and they are not willing to try." Additionally, participants expressed concerns regarding students with learning disabilities, students that participant 6 refers to as *our kids.*

Participant 6: I thought of our kids. I am always thinking of our kids.

Participant 7: Yes, what direction they are going in.

Participant 6: They are so different. They do not see math or have a vision of what is happening. Our kids need to see something more times.

Late SAPD program. Primary sources of qualitative data representing participants' pedagogical beliefs in the final stages of the SAPD program include SAPD session recordings, implementation feedback forms, and post-intervention semi-structured interviews. Although cohesive beliefs cannot be inferred from isolated comments, participant responses help to explain the significant change in overall pedagogical beliefs (p = .017). Distinct changes and stagnations in beliefs are described in the following paragraphs.

The final item of the semi-structured interview prompted participants to reflect on their experience as a participant in the SAPD program. The response of participant 2 indicates a notable change in beliefs.

> I like less of a 'here is what you have to do' approach. I like (the students) to come up with concepts and develop them on their own. I want to continue to do this. To guide [the students] to the ideas and principles that are important for the concepts as opposed to just telling them. (Participant 2)

Although this participant's beliefs were not recorded in quantitative measures, this response indicates a change in teaching beliefs, similar to the mean change of the other participants (M = 2.88; p = .038).

Qualitative data provide evidence of change in pedagogical beliefs for participant 9. Quantitative findings indicate that participant 9 experienced one of the largest increases in MTPDBI scores from pre- to post intervention (44 and 58 respectively). This participant's responses during SAPD sessions and on implementation feedback forms support this increase.

When reflecting on an SAPD session activity involving the use of correct and incorrect worked examples to teach problem-solving, participant 9 noted that having students explain processes helps them internalize rather than memorize the process. In addition, participant 9 disagrees that showing students incorrect examples confuses them, rather participant 9 believes "it makes them think."

Qualitative findings from the early and middle stages of the SAPD program reflect participant concerns that reform-minded instruction usurps time from instruction that could otherwise be devoted to the practice and reinforcement of procedural skills. Data from final implementation forms reveal that participants continue to measure student success in terms of the time required for students to master mathematics procedures and skills. However, consistent with Guskey's (2002) assertion that "successful implementation that changes teachers' attitudes and beliefs' (p. 383), participants expressed a change in attitude toward reform-minded instruction after the experience positive student outcomes when implementing reform-minded lessons. Participants made the following remarks after individual instructional coaching sessions implementing the strategy of analyzing correct and incorrect worked examples:

Participant 9: I truly feel this was the fastest my kids have ever learned this technique [referring to factoring by completing the square]. Great lesson.

Participant 2: My students passed their sequence test with flying colors!

Data from final implementation feedback forms and responses to semi-structured interviews illustrate how general teaching efficacy toward students who struggle in mathematics remains a barrier to adopting learner-centered beliefs. General teaching efficacy refers to an educators' belief in the ability of students to learn (Ashton & Webb, 1986). Participants 6 and 7 express concerns with implementing reform-minded instruction with students with disabilities. During a semi-structured interview, participant 6 stated "some of the higher ideas did not seem to directly apply to my students. I would have to adjust the activities with my learners that have academic challenges." Similarly, on an implementation feedback form, participant 7 questioned:

> How can I help students with deficits in working memory, processing speed, fluid reasoning skills, and math reasoning [not just participate] but actually LEARN from participating in activities that require these as pre-requisite skills? (Participant 7)

These immunities to change may help to explain why quantitative measures of pedagogical beliefs under the subscales of learning, discourse, and nature of math did not significantly change.

Qualitative data illustrate individual participants' changes in beliefs regarding reform-minded instruction throughout the three stages of the SAPD program. A summary of qualitative responses indicative of participants' instructional beliefs at three points throughout the SAPD program is provided in Table 5.8.

Table 5. 8

Summary of Participants' Description of Reform-Minded Instructional Beliefs at Three Points in SAPD Program

Time	Participant Response Indicating Beliefs	MTPDMI Theme
Early SAPD Program	"Kids would be more successful in math if they [were] drilled more in basic facts" (Participant 9)	Teacher-Centered
	"All of us learned algebra and geometry that was [largely] traditional and we all turned out ok." (Participant 2)	
	"we can't spend a lot of time on [reform-minded lessons] because there is so much we have to teach." (Participant 9)	
	"Some of the aspirations I have for [students with learning disabilities] are not necessarily those I share for typical students." (Participant 7)	
	"I think that to prepare kids for where they are going to be today, we need to focus more on [concepts] than specific [procedures]." (Participant 1)	Emerging
	I don't think (end-of-course exam) questions are homing in on random skills. The test is trying to get them to problem solve…[they] are going to need to know how to problem solve." (Participant 3)	
Mid SAPD Program	"You don't have to be a problem solver to get the questions right. If you teach a tested subject, you are judged by [whether or not] your [students] get the questions right." (Participant 5)	Teacher-Centered
	"[Our kids] do not see math or have a vision of what is happening. Our kids need to see something more times." (Participant 6)	

Table 5.8 (continued)

	"I am always torn between mechanics and context. Part of me says you need to know how to divide polynomials but another part says with technology being what it is calculators will be able to do that stuff for [the student]. In terms of error analysis, I like [this strategy] because [the students] can think about the misstep. [The students] become the teacher. It's that idea of how much we should focus on mechanics." (Participant 2) "[Students] are not used to reform-minded teaching. [These students] do not know what to do and they are not willing to try." (Participant 3)	Emerging
	"It's not about showing [the students] different ways, but really trying to help them see connections between everything. "(Participant 3)	Approaching
	"With reform-minded lessons, kids can take you in another direction. They take longer than you think. You have to be willing to adapt within the lesson itself." (Participant 1)	Learner-Centered
Late SAPD Program	"Some of the higher [reform-minded] ideas did not seem to directly apply to my students. I would have to adjust the activities with my learners that have academic challenges." (Participant 6)	Teacher-Centered
	"How can I help students with deficits in working memory, processing speed, fluid reasoning skills, and math reasoning [not just participate] but actually LEARN from participating in [reform-minded] activities that require these as pre-requisite skills? (Participant 7)	Emerging
	I" like less of a 'here is what you have to do' approach. I like (the students) to come up with concepts and develop them on their own. I want to continue to do this. To guide [the students] to the ideas and principles that are important for the concepts as opposed to just telling them." (Participant 2)	Learner-Centered

Implementation of Reform-Minded Instructional Practices (RQ3)

In this section, I examine changes in participants' instructional practices over the course of the SAPD program (RQ3). Participants were observed using the RTOP pre- and post-intervention. Quantitative analysis includes descriptive and inferential statistics of overall RTOP scores as well as on the subscales of lesson plan and implementation, propositional knowledge, procedural knowledge, classroom culture, and student-teacher relationships. A qualitative analysis of SAPD session recordings, participant implementation feedback forms, and researcher field notes will provide evidence for analysis.

To determine if participants' use of reform-minded instructional practices changed throughout the duration of the SAPD program, RTOP scores from pre- and post-intervention classroom observations were compared. Reliability and validity of the RTOP have been established through reports of interrater reliability (Sawada et al., 2002), face validity (Sawada et al., 2002), and predictive reliability (Ellis et al., 2007). Authors of the RTOP report 98% interrater reliability (Sawada et al., 2002) while researchers Ellis et al. (2007) report interrater reliability scores ranging from 70% to 89% for each of the five subscales of the RTOP. An overall Cronbach's alpha score of .97 confirms the instrument's reliability as a measure of reform-minded instruction (Sawada et al., 2002).

The RTOP (Sawada et al., 2002) assesses the level of reform-minded instruction present in mathematics lessons. Using the RTOP, classroom observers indicate the prevalence of 25 reform-minded characteristics on a scale from 0 to 5. A score of 0 indicates that the characteristic was not present, while a score of 5 indicates that the characteristic was indicative of the lesson (Sawada et al., 2002). Overall observation scores range from 0 to 100 and categorize instruction into five levels, dependent on levels of reform-mindedness (Table 5.9). RTOP items are divided into 5 subscales; lesson design and implementation, propositional pedagogical knowledge, procedural pedagogical knowledge, communicative interactions, and student-teacher relationships. Possible scores for each subscale range from 0 to 20. Participants pre-intervention and post-intervention RTOP scores, along with corresponding category ratings, are included in Table 5.10.

Differences in participants' pre-intervention and post-intervention RTOP scores were analyzed for normalcy.

Table 5. 9.

RTOP Levels: Categorized by Score and Type of Teaching (Ebert-May et al., 2011)

RTOP Level	RTOP Score	Type of Teaching
I	0 - 30	Straight lecture
II	31 - 45	Lecture with some demonstration and minor student participation
III	46 - 60	Significant student engagement with some minds-on as well as hands-on involvement
IV	61 - 75	Active student participation in the critique as well as the carrying out of experiments
V	76 - 100	Active student involvement in open-ended inquiry, resulting in alternative hypotheses, several explanations, and critical reflection

Table 5. 10

Summary of Pre-Intervention and Post-Intervention RTOP Scores and Type by Participant

	Pre-Intervention RTOP Data		Post-Intervention RTOP Data	
	Score	Type	Score	Type
Participant 1	30	I	44	II
Participant 2	25	I	27	I
Participant 3	28	I	56	III
Participant 4	20	I	35	II
Participant 5	32	II	27	I
Participant 6	25	I	33	II

Table 5.10 (continued)

Participant 7	28	I	31	II
Participant 8	22	I	37	II
Participant 9	30	I	33	II

A test for normalcy indicates that the distribution of differences in participants' RTOP scores is moderately skewed (skewness = .569; kurtosis = .380). Because the normalcy assumption was not met, the nonparametric Wilcoxon Signed Ranks test was used to compare pre- and post-intervention RTOP scores. The change in participants' RTOP scores from pre- to post-intervention was found to be significant ($p = .028$). The mean difference between pre-intervention (M = 26.27; SD = 3.97; N = 9) and post-intervention (M = 35.89; SD = 9.16; N = 9) RTOP scores was 9.62, indicating movement away from lecture as a primary source of instruction. Changes in scores for the lesson design and implementation and procedural pedagogical knowledge subscales were also significant ($p = .020$ and $p = .017$ respectively). Changes for the propositional pedagogical knowledge, communicative interactions, and student-teacher relationships were not significant ($p = .103$; $p = .137$; $p = .438$ respectively). An outline of descriptive and inferential findings is included in Table 5.11.

Table 5. 11

Summary of Descriptive and Inferential Statistics: Measure of Implementation of Reform-Minded Instructional Practices

Reformed Teaching Observation Protocol (Sawada et al., 2000)	Pre $N = 9$		Post $N = 9$		Wilcoxon Signed Ranks Test	
	M	SD	M	SD	*z*	*p*
Total Score	26.27	3.97	35.89	9.16	-2.196[b]	.028*
Subscales						
Lesson Design and Implementation	3.11	.93	5.67	2.54	-2.322[b]	.020*
Propositional Pedagogical Knowledge	7.33	2.55	8.56	2.00	-1.630[b]	.103
Procedural Pedagogical Knowledge	4.00	1.41	6.56	2.60	-2.393[b]	.017*
Communicative Interactions	4.22	2.05	6.00	1.59	-1.489[b]	.137
Student-Teacher Relationships	8.00	1.5	9.11	3.85	-.775[b]	.438

Sources of qualitative data assessing participants' implementation of reform-minded instructional practices include field notes from classroom modeling sessions, participant implementation feedback forms, semi-structured interviews, and researcher notes on RTOP observation documents. As with RQ1, qualitative data were coded and categorized into the RTOP themes of *lesson design and implementation, propositional pedagogical knowledge, procedural pedagogical knowledge, communicative interactions,* and *student-teacher relationships.* Along with the SAPD program foci a priori codes, several emergent codes were revealed. Table 5.12 provides a summary of a priori and emergent codes, organized by RTOP.

Table 5. 12

A Priori and Emergent Codes Aligned with RTOP Subscales: Qualitative Indicators of Implementation of Reform-Minded Instructional Practices

RTOP Subscale Theme	A Priori Codes	Emergent Codes
Lesson Design and Implementation	Prior knowledge Student exploration	Independent practice Guided practice Lecture
Propositional Pedagogical Knowledge	Conceptual understanding	Procedural focus Fundamental concepts Real world connection
Procedural Pedagogical Knowledge		Multiple Representations Clarification Intellectual rigor
Communicative Interactions	Student Discourse	Questioning
Student-Teacher Relationship	Teacher as facilitator	Teacher directed

Effective educator professional development programs seek to improve student learning through improved instruction (Learning Forward, 2011; Lieberman & Miller, 2014). Reform-minded mathematics instructional practices represent an improvement over traditional instructional practice and have been endorsed by national educational organizations and experts (National Mathematics Advisory Panel, 2008; NCTM, 2014). Pre-intervention RTOP data indicate that participants predominantly implement traditional practices (M = 26.17; SD = 3.97; N = 9) illustrating a need for instructional improvement. Instructional change occurs slowly as a result of contributing factors (Clarke & Hollingsworth, 2002). Therefore, the following analysis presents indicators of changes in participants' instructional practices at three stages in the SAPD program.

Early SAPD program. Early classroom observations indicate that participants designed and implemented lessons using lecture and guided practice as a main source of instruction.

Lecture and guided practice oppose the lesson design and implementation elements characteristic of reform-minded instruction (Sawada et al., 2002). The predominant use of lecture and guided practice (66.6%; $N = 9$) offers a possible explanation for participants' low pre-intervention score for this RTOP subscale ($M = 3.11$; $SD = .93$; possible subscale range 0 – 20; $N = 9$). Researcher notes on RTOP observation forms include comments that classroom teachers worked examples on the board while students filled in notes during the majority of observations (66.6%; $N = 9$). Additionally, participant 1 had students working independently through geometry curriculum on a computerized, self-paced program. Almost half of the participants (44.4%: $N = 9$) began classroom instruction with a review of prior knowledge, a core element of reform-minded lessons (Sawada et al., 2002). However, similar to the design of the lesson, reviews of prior knowledge consisted of "bell-work" (Participant 4) or "bell-ringers" (Participant 3) involving student completion of procedural tasks from the previous day's lesson. Discussion of prior knowledge was limited to individual student questions regarding procedures. Participants' procedural focus during mathematics instruction provides a possible explanation for the low pre-intervention score for the lesson design and implementation subscale ($M = 3.11$; $SD = .93$; possible subscale range 0 – 20; $N = 9$).

Hallmarks of reform-minded mathematics instruction include focusing on fundamental concepts within mathematics courses and maintaining intellectual rigor (Sawada et al., 2002). Although most of the participants' observed lessons focused on fundamental concepts of their respective courses, evidence of intellectual rigor was lacking. Consistent with participants' low mean procedural pedagogical knowledge subscale score ($M = 4.00$; $SD = 1.41$; possible score range 0 – 20; $N = 9$), research notes from pre-intervention observations indicate that participants' lessons lack elements that challenge learners to extend their mathematical thinking.

Illustratively, when learning to solve quadratic equations by factoring, students were provided with charts that included factors of common numbers and were directed to "use the table" (Participant 2) in lieu of exploring patterns. Additionally, participants 7 and 9 directed students to "use the a-b-c key" on the calculator when computing with fractions." Participants 2 and 9 also reminded students to take detailed notes because, "whatever you right down you can use on tests and quizzes" (Participant 9).

Reform-minded mathematics instruction relies on student-to-student discourse (Eddy et al., 2015; Sawada et al., 2002). RTOP observation documents contain evidence of variation among participants regarding the use of student discourse. Participants 5, 6 and 7 implemented lessons where students were placed into collaborative groups. Although group tasks largely focused on procedures and skills, students were observed discussing feasibility of solutions and assisting peers in the completing of multi-step procedures. Student discourse for the remaining participants was limited to question and response discourse (Lischka & Garner, 2016). Commonly, participants involved students in discourse by pausing when modeling problems and allowing students to volunteer solutions. Consistent implementation of teacher discourse rather than student discourse may explain the low, mean communicative interactions subscale score (M = 4.00; possible subscale range 0 – 20; N = 9) for participants.

Qualitative data for pre-intervention observations suggest that participants' mathematics instruction largely focuses on procedural rather than conceptual knowledge, a measure of propositional pedagogical knowledge (Sawada et al., 2002). Observation records indicate that the primary task for students in most classrooms (66.6%; $N = 9$) was to complete a set number of problems focusing on procedural skills. For example, participant 5 asked students to solve 32 multi-step algebraic equations prior to the end of class. The students of participant 4 were asked to complete a handout of problems involving writing linear equations given slope and y-intercept. Similarly, after reviewing the procedure for solving quadratic equations, participant 2 asked students to complete six problems independently. A focus on procedural tasks for many participants may explain low overall and propositional pedagogical knowledge subscale mean scores.

Observation data also suggest that participants generally implement teacher-directed lessons rather than facilitating lessons guided by student conjectures, questioning, and strategizing (Sawada et al., 2002). Participants' mean pre-intervention score of 8.00 (possible subscale range 0 – 20; $N = 9$) for the student-teacher relationship subscale confirms the prevalence of teacher-centered instruction. Conceivably, student-directed, or student-centered lessons would be open-ended in that the lessons would support continued investigations and the seeking out of alternative solutions (Lischka & Garner, 2016; Sawada et al., 2002). Conversely, teacher-directed lessons would be predetermined and finite. Qualitative data suggest that the majority (88.9%; N = 9) of participants' lessons were teacher-directed.

As noted earlier, participants' mathematics lessons largely involved the modeling and repetitious completion of procedural skills, such as solving quadratic functions by factoring (Participant 2). On all occasions, individual instruction was complete when pre-determined problems were complete. Furthermore, students were directed toward non-mathematics tasks (e.g., using cellular devices) upon completion of the assigned problems. Students were observed engaging in non-mathematics, non-school tasks as with as much as seven minutes (Participant 4) and 15 minutes (Participant 2) of class time remaining, which helps to explain the low pre-intervention RTOP scores for these participants (25 and 20 respectively).

Mid SAPD program. Following the initial three SAPD sessions, participants began infusing learned reform-minded instructional strategies into classroom lessons. Although implementation of learned reform-minded strategies was a design component of the SAPD program, choice of reform-minded elements was determined by participants. Data collected from participant implementation feedback forms, along with field notes from classroom modeling and coaching sessions indicate variation among participants in implementing reform-minded instructional practices at the midpoint in the program.

Participant implementation feedback forms provide evidence that most (66.7%; N = 9) participants began implementing lessons involving student exploration. Participants report using Desmos Classroom Activities (Desmos, 2019) as an effective avenue for student exploration of geometric translations, parameters of trigonometric equations, graphs of quadratic functions, and transformations of logarithmic functions. Participant 1 engaged students in a Transformation Golf (Desmos, 2019) activity and reported that students "struggled with rotations [at first], but [eventually] figured it out." Participant 1 indicated a desire "to make some of my own [classroom activities]." Likewise, participant 9 described an experience with the Card Sort: Parabolas (Desmos, 2019) activity as "positive and supportive" and indicated that "[I] would use it again." Of the six participants who implemented Desmos Classroom Activities (Desmos, 2019), five participants specifically reported positive student feedback which may contribute to continued implementation.

In reform-minded instruction teachers become facilitators and listeners rather that directors (Sawada et al., 2000). Data from implementation feedback forms also suggest that participants began to (a) adopt the role of facilitator and (b) purposefully infuse student discourse into mathematics lessons. During this stage of the SAPD program, participants implemented lessons requiring them to serve as facilitators. Participants described being "available for clarification, not answers" (Participant 8), directing students without "[telling them] exactly how to do it" (Participant 1), and appreciating as "students caught on and helped out each other" (Participant 3). Implementation feedback forms also illustrate that participants began implementing lessons requiring students to "explain their reasoning for solutions" (Participant 5) and "summarize main points of the lesson" (Participant 7), activities that increase student discourse. Feedback forms indicated that students "were not used to explaining" (Participant 5) and preferred "being told exactly how [the outcome] should look" (Participant 1) providing evidence of changes in participant' instructional practices. Although participants began to envision the role of instruction differently, evidence suggests that the role of students did not change. Rather than investigating or exploring divergent thinking (Sawada et al., 2000), students collaborated on problems involving closed-ended solutions.

Reform-minded instruction encourages the use of multiple representations to demonstrate understanding of mathematics content (Sawada et al., 2002), yet little evidence exists to suggest participants are employing this strategy. In fact, qualitative data from mid SAPD session feedback forms support that only two of the nine participants implemented lessons involving the specific use of multiple representations. On two occasions, Participant 7 reinforced mathematics procedures by having "students demonstrate solutions with manipulatives" and encouraging students to "create illustrations to break apart monomials and polynomials." Data from implementation feedback forms suggest that some participants continued to focus on procedural rather than conceptual understanding. Participants 2, 3, and 5 implemented lessons that relied on student-to-student discourse, but focused on solving procedural algebraic or geometric problems. For example, participant 3 reflected on collaborative lessons where "students worked in groups of four [to complete] a surface area and volume puzzle." Students "worked together to find the solutions" (Participant 3), yet the solutions appeared to be obtained simply by inserting given dimensions into formulas. Conversely, participant 1 reflected on an implemented lesson where students applied knowledge of conic sections on a "design your own school project" (Participant 1). Qualitative data suggest variation in propositional pedagogical knowledge among participants.

Late SAPD program. Increased RTOP ratings, by one category level, positively impacts student outcomes (Holt et al., 2015). Most participants in this study (77.8%; N = 9) experienced increases in RTOP scores resulting in movement to a higher category type (Ebert-May et al., 2011), as indicated in Table 5.8. Qualitative data from post-observation RTOP documentation provide justification for changes in RTOP scores by subscale.

A Wilcoxon Signed Ranks test indicates that changes in mean RTOP scores for the lesson design and implementation subscale from pre- to post-intervention were significant (p = .020). Although participants improved on lesson design and implementation subscale items, the post-intervention mean score of 5.67 (mean score range 0 – 20; N = 9) remains the lowest of all post-intervention RTOP subscale scores. Lesson design and implementation indicators include the promotion of student exploration, instruction that responds to students' prior knowledge, and high levels of student engagement (Sawada et al., 2002). Qualitative data, collected from RTOP observation documents, indicate areas where participants' instructional practices changed or remained the same.

During post-intervention lessons, participants began to infuse student exploration into mathematics instruction which contributed to the significant change in mean score for the lesson design and implementation subscale (M_{diff} = 2.56; SD_{diff} = 1.61; n = 9; p = .020). In a lesson focusing on finding the area of composite figures, participant 6 encouraged students to consider "how else the [composite] shape could be divided?" Participant 6 inquired, "could we do this a different way?" and "is this the only [possibility]?" then allowed students time to brainstorm. Similarly, in a geometry lesson on constructing angles, participant 3 began stated "I want you to explore and [then] explain how to [inscribe] a square in this circle," before modeling the correct procedure. Participant 7 also infused exploration by introducing students to the *trace* feature on graphing calculators. Participant 7 encouraged students to explore linear functions by asking "what do you notice?" as students followed the screen's cursor. The introduction of student exploration into mathematics lessons may help to explain the increase in overall RTOP score from pre- to post-intervention for participants 3 (from 28 to 56), 6 (from 25 to 33), and 7 (from 28 to 31).

Observation documents indicate that more than half of participants (55.6%; N = 9) began instruction by reviewing prior knowledge or students' misconceptions, providing support for the significant increase in lesson design and implementation subscale score ($M_{\text{diff}} = 2.56$; $SD_{\text{diff}} = 1.61$; $n = 9$; $p = .020$). While participant 2, who remained at RTOP level I throughout the SAPD program, and participant 9, who moved from level I to level II, asked students to recall procedural tasks from prior lessons while working out an example on the board, participants 3 and 4 engaged students in hands-on activities that required the application of prior knowledge. Participant 3, for example, asked students to work with partners to recall procedures for constructing angles of various sizes using a protractor. Participant 3 walked around the room offering assistance to pairs of students needing support and addressed the whole class when the need arose as indicated by the following: "Now that I have refreshed your memory of how to construct angles, I want you to [construct] a 55° angle" (Participant 3). Student misconceptions also guided a review of graphing linear equations conducted by participant 4. While observing students as they worked, participant 4 interjected "I see that you are identifying the *y*-intercept, but first we have to…," correcting students' misunderstanding before continuing. The implementation of lessons that respond to students' prior knowledge contributed to changes in RTOP levels for participants 3 and 4 (from I to III and I to II respectively).

Although elements of student exploration and accessing prior knowledge emerged in these final observations, many participants continue to favor traditional forms of mathematics instruction. In five out of nine observations, participants guided students through content while lecturing or working examples at the board while students copied the worked examples in their notes. Even when students were engaged with partners or in small groups, as with participants 1, 4, and 5, students relied on each other for verifying solutions rather than for learning content. Reliance on teacher-centered, traditional forms of instruction contributed to non-significant changes in mean scores for the propositional pedagogical knowledge ($M_{\text{diff}} = 1.23$; $N = 9$; $p = .103$) and communicative interactions ($M_{\text{diff}} = 1.78$; $N = 9$; $p = .137$) subscales. Continued reliance on traditional, teacher-directed instruction could result from participants' focus on preparing students for end-of-course exams. The majority of participants (66.6%; $N = 9$) teach courses associated with end-of-course exams. Research indicates that teachers faced with high-stakes exams may resort to teacher-directed lessons in an attempt to prepare students for these exams (Au, 2007).

Qualitative data from post-intervention RTOP observations support these findings and provide a possible explanation for the lack of significant change. Post-intervention observation documents indicate that most participants (66.6%; $N = 9$) designed lessons around fundamental concepts of participants' respective courses. In these instances, lesson objectives aligned with state content standards, but continued to focus on procedural rather than conceptual understanding. Participant remarks during post-intervention observations illustrate a focus on procedural tasks. In a lesson on solving quadratic equations by completing the square participant 9 stated, “I am going to give you the steps” while writing them on the board. Furthermore, when a student requested help, participant 9 stated “Let’s look at the steps” as the student was guided through the procedure. Similar interactions occurred with participants 2 and 5. When teaching students to evaluate sequences, participant 2 reminded the students that “if you make a mistake, you probably put it in the calculator wrong” and relayed to the observer “these students need structure.” Furthermore, in a lesson on factoring quadratic equations, participant 5 not only focused on procedures, but focused on calculator key strokes to “reverse engineer” (Participant 5) binomial factors. As indicated by post-intervention RTOP data, participant 2 remained at RTOP level I (Ebert-May et al., 2011) upon completion of the SAPD program while participant 5 dropped to level I (from level II).

Research indicates that educators respond to professional development programs at varying rates (Desimone & Garet, 2015). A comparison of pre- and post-intervention RTOP scores and categories by participant (Table 5.9) indicates that a response variance exits for this group of participants. Qualitative findings help to explain varying rates of responsiveness and in particular the drop in RTOP score and category for Participant 5. RTOP observation notes indicate that participant 5 teaches in a high-stakes context (Gebril & Eid, 2017). Teachers responsible for high-stake exams resort to traditional forms of instruction to cover tested content (Au, 2007; Gebril & Eid, 2017). As noted in earlier quantitative analysis, participant 5 expressed explicit concerns regarding instructional possibilities “if you did not have to get ready for the [end-of-course] test” (Participant 5).

Observation documents indicate small changes in propositional pedagogical knowledge for some participants. Evidence of real-world applications appears in observation notes for some participants (33.3%; N = 9). Participant 1 designed a lesson requiring students to apply probability rules to real-world scenarios. Participant 2 described how sequences are used in computer programming. Finally, participant 6 built a lesson on calculating area of composite figures around the scenario of carpeting the first level of a home. Even minor changes in instructional practices, such as introducing real-world scenarios into mathematics lessons, lead to increases in RTOP scores and even small increases in RTOP scores indicate movement toward reform-minded instruction (Ebert-May et al., 2011). The small changes propositional pedagogical knowledge contributed to significant ($p = .023$) changes in the mean RTOP score for this subscale.

A Wilcoxon Signed Ranks test reveals a significant difference ($p = .017$) between pre-intervention mean score ($M = 4.00$; $SD = 1.41$; $N = 9$) and post-intervention mean score ($M = 6.56$; $SD = 2.60$; $N = 9$) for the procedural pedagogical knowledge subscale. Qualitative data confirm these findings and indicate that procedural pedagogical knowledge was an area of improvement for participants. Observation documents indicate participants' lessons increased in rigor, compared with pre-intervention data, and began to encourage students to clarifying their thinking, characteristics of procedural pedagogical knowledge (Sawada et al., 2002). Participant 1 engaged students in a cooperative activity requiring the application of probability rules. "You are realizing that some of the tasks are designed to push your thinking" stated participant 1 who recognized that "some of the tasks are a bit more challenging" (Participant 1). Participants questioned and prodded students with "how will we know?" (Participant 7), "walk me through it" (Participant 4), and "why did that work?" (Participant 3) indicating an interest in student clarification and explanation.

Although not an area of significant change ($M_{\text{diff}} = 1.11$; $N = 9$; $p = .438$), the student-teacher relationships subscale is an overall area of strength for this group of participants. The mean post-intervention score of 9.11 (mean range 0 – 20; $N = 9$) indicates that participants sought student engagement and adopted the role of facilitator in the classroom. Compared with pre-intervention data indicating a predominant use of teacher-directed instruction by participants (88.9%; $N = 9$), post-intervention documentation indicates that almost half (44.4%; $N = 9$) of participants shifted away from teacher-directed instruction and toward a student-directed approach. Teachers took on a facilitative role and either (a) responded to the needs of small groups or pairs of students directly or (b) periodically interrupted discourse to clarify procedures or misconceptions as they arose. In spite of participants more frequently taking on the role of facilitator in post-intervention lessons, the content of each lesson was predetermined by participants and students were directed in their learning along convergent, rather divergent, methods of problem solving.

Consistent with quantitative findings suggesting that significant ($p = .028$) changes occurred regarding the implementation of reform-minded instructional practices, qualitative data reveal that participants attempted to infuse reform-minded strategies into mathematics lessons. Although many participants' (55%; $N = 9$) lessons were structured around direct instruction, the implementation of reform-minded strategies was evident in the final stages of the SAPD program. Table 5.13 Highlights the self-reported and observed instructional practices participants implemented throughout the SAPD program illustrating the variance in participants implementation of reform-minded instructional practi

Table 5. 13

Summary of Self-Reported and Observed Instructional Practices at Three Points in SAPD Program

Timeframe	Instructional Strategies	RTOP Subscale
Early SAPD Program	Teachers worked examples on the board while students filled in notes	Lesson Design and Implementation
	Students completed bell work as a way of reviewing prior knowledge	
	Students worked independently through geometry curriculum on a computerized, self-paced program	Propositional Pedagogical Knowledge
	Students were taught to "use the a-b-c key" on the graphing calculator to compute with fractions	Procedural Pedagogical Knowledge
	Students engaged in notetaking and were reminded that notes could be accessed during assessments	
	Teachers promoted question and response discourse	Communicative Interactions
Mid SAPD Program	Use of Desmos Classroom Activities (Desmos, 2019)	Lesson Design and Implementation
	Collaborative lessons focused on completing procedural tasks	Propositional Pedagogical Knowledge
	Students required to explain their reasoning	Procedural Pedagogical Knowledge
	Students demonstrated solutions with manipulatives	
	Student created illustrations to model problems	
	Students collaborated on closed-end problems	Student-Teacher Relationships
	Teachers available for clarification, not answers	
Late SAPD Program	Teachers lectured and worked examples at the board	Lesson Design and Implementation
	Teacher introduced trace key to students and allowed them to explore properties of given functions	

Table 5.13 (continued)

	Lesson included real-world references and applications Teachers reinforced steps necessary to solve procedural problems	Propositional Pedagogical Knowledge
	Teachers' questioning included "Why did that work?" and "How do we know?" requiring students to clarify	Procedural Pedagogical Knowledge
	Teachers served as facilitators, pausing collaborative work to interject when needs arose	Student-Teacher Relationships

The purpose of this study was to investigate the impact of the SAPD program participants' knowledge, beliefs, and implementation of reform-minded instructional practices. Although the small sample size used in this study impacts the ability to determine a causal relationship (Shadish et al., 2012), study results indicate a significant (p = .017) change in participants' beliefs regarding reform-minded instruction and a significant (p = .028) change in participants' implementation of reform-minded instructional practices throughout the duration of the year-long SAPD program. Qualitative findings support a change in participants' beliefs and instructional practices and provide a deeper understanding of changes over time. As supported by quantitative and qualitative findings, participants' knowledge of reform-minded instructional practices remained unchanged. Quantitative findings confirm the complexity of educator growth and development (Clarke & Hollingsworth, 2002) and the factors impacting change (Cross, 2009; Guskey, 2002).

Process Evaluation

In this section, I will examine the process evaluation questions. Process evaluations examine the behaviors of program participants and organizers and answer questions regarding program effectiveness (Baranowski & Stables, 2000). The process evaluation involves an examination of the features of the SAPD program that participants describe as most impactful and least impactful in transforming pedagogical beliefs and instructional practices. I will also be examining the fidelity of implementation components of dose delivered, dose received, program adherence, and participant responsiveness.

Assessment of SAPD Program Components

Research questions four and five intend to explore the SAPD program elements that participants found most impactful and least impactful to changes in knowledge, beliefs, and implementation of reform-minded mathematics instruction. This examination relies on a quantitative analysis of post-intervention survey results and a qualitative analysis of participant responses to post-intervention semi-structured interviews.

Most and Least Meaningful SAPD Program Aspects (RQ4 and RQ5)

To determine participants' perceptions of the SAPD program components, mean ratings on the customized Panorama survey were calculated. The Panorama Teacher Survey (Panorama Education, 2015) is a compilation of surveys intending to measure teachers' perceptions on a variety of topics. The custom survey used in this research (see Appendix E) contains items measuring participants' overall perceptions of the SAPD program as well as participants' perceptions of the SAPD classroom modeling component. Panorama survey developers recognize the importance of validity and reliability reporting and report data are currently being collected to obtain these measures (Panorama Education, 2015).

The adapted Panorama Professional Learning Survey (Appendix E) includes eight Likert scale items. Each item measures a different aspect of the professional development and responses range from *not* to *extremely* present based on the construct measured in each item. For example, when asked how supportive the SAPD program provider was, the Likert scale ranged from *not at all supportive* to *extremely supportive.* Remaining items followed a similar scale. Each response was quantified on a scale from 1 (not present) to 5 (extremely present). Mean scores were calculated and qualified based on survey descriptors. Overall ratings corresponded the Likert descriptors based on the mean score for each item. Table 5.14 outlines the adapted survey results. Quantitative data reveal that participants found the program supportive, valuable, helpful and relevant and that participants learned many strategies and had some input into SAPD program content.

Table 5. 14

Summary of Panorama Survey Data: Indicators of Participants' Perceptions of SAPD Program Aspects

		Range		
Survey Item	Mean	Minimum	Maximum	Overall Rating
How supportive has this professional development provider of your growth as a teacher?	4.8	4	5	Quite supportive
How valuable was this professional development opportunity?	4.7	4	5	Quite valuable
How helpful were your colleagues' ideas for improving your teaching during this program?	4.3	3	5	Quite helpful
How helpful was this professional development opportunity in helping you explore new ideas?	4.8	4	5	Quite helpful
How relevant has this professional development opportunity been to the content that you teach?	4.2	3	5	Quite relevant
Though this program, how many new teaching strategies have you learned?	4.8	2	5	Many strategies
How much input did you have into individualizing this professional development opportunity?	3.7	1	5	Some input
How much did you learn about teaching from this professional development?	4.6	4	5	Learned quite a bit

Participants gave the highest mean ratings (*M* = 4.8; range 1 – 5; *N* = 9) to survey items regarding the support offered by the facilitator, the quantity of strategies learned in the program, and the opportunities within the SAPD program for exploration. Additionally, the mean ratings on items reflecting the value of the SAPD program and the amount of personal learning that occurred were of 4.7 and 4.6 respectively (range 1 – 5; *N* = 9). The survey item with the lowest mean score (*M* = 3.7; range 1 – 5; *N* = 9) referred to the amount of input participants had in individualizing the SAPD program. This item also elicited responses with the greatest range. Six participants indicated that the program afforded them *quite a bit of input,* one participant responded with *a tremendous amount of input,* one participant responded with *some input,* and one participant indicated that the program allowed *almost no input.* The variation in participants' rating of the SAPD program components can be explained by a qualitative analysis of post-intervention semi-structured interviews.

Following a directed approach to content analysis, the core features of effective professional development; *situated, learner-center, collaborative,* and *supportive* (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Fisher & Frey, 2014; Garet et al., 2001), form the themes for qualitative data analysis. Participant responses were coded under the a priori codes of *context embedded, active engagement, whole group discussions, teacher-based-team discussions, modeling,* and *coaching.* The codes of *applicable, specific instructional strategies, experiential, like-mindedness, team teaching, self-reflection, duration, meeting time, planning time, facilitator,* and *ownership* emerged during the data analysis process. A display of themes and related codes can be found in Table 5.15. The following qualitative analysis seeks to address research questions four and five and is organized according to the features that participants found most effective and least effective in in terms of developing participants' knowledge, beliefs, and use of reform-minded instructional practices.

Table 5. 15

A Priori and Emergent Codes Aligned with SAPD Program Aspects: Qualitative Indicators of Participants' Perception of SAPD Program

Theme	A Priori Codes	Emergent Codes
Situated	Context embedded	Applicable Specific instructional strategies
Learner-centered	Active engagement	Experiential
Collaborative	Whole-group discussions TBT discussions	Like-mindedness
Supportive	Modeling Coaching	Team teaching Self-reflection
Time		Duration Meeting time Planning time
Leadership		Facilitator Ownership

Most meaningful (RQ4). Qualitative data, collected from post-intervention semi-structured interviews, provide an explanation for participants' positive perceptions of the SAPD program as indicated by generally high ratings on the Panorama Teacher Survey (Panorama Education, 2015). Participant responses during individual face-to-face interviews pinpoint the components of the SAPD program that participants found most meaningful in their transformation toward reform minded teaching. Components that participants found valuable fall under the themes of situated, learner-centered, collaborative, supportive, time, and leadership. The prevalence of these themes in participant responses indicate that participants valued the sociocultural approach inherent in the design of the SAPD program. Indicative of sociocultural theory (Vygotsky, 1978), participants attributed personal growth to their interaction with peers.

Qualitative data indicate that participants preferred the situated context of the SAPD program, supporting a mean score of 4.2 (mean score range 1 – 5; $N = 9$) on the Panorama survey relevancy item. Participants described the content of the SAPD program as “more applicable to my exact field and discipline and what I do in my classroom” (Participant 5). Participant 4 also found the content directly applicable and stated; “I felt like I could actually apply it to the way that I teacher rather that substituting in for the way I teach.” Participant 1 described that the SAPD content “was much more applicable in that it gave specific strategies and specific research based methods to help in the classroom.”

Participants also noted that, compared with other professional development, the SAPD program provided them with specific mathematic strategies to implement in the classroom. Quantitative data indicate that participants learned many strategies ($M = 4.8$, mean score range 2-5; $N = 9$) throughout the duration of the SAPD program. When asked to identify program activities they enjoyed the most, participants stated that they enjoyed being introduced to specific reform-minded instructional strategies including Desmos Classroom Activities (Desmos Inc., 2019). Participant 5 identified “learning about the instructional [capabilities] of Desmos” as the most enjoyable SAPD program activity as did Participant 9. Participant 4 stated that “learning about Desmos” contributed to a change in instructional practices. The desire to continue to use Desmos Classroom Activities (Desmos, 2019) was expressed by participant 2 who identifies as “a Desmos fan.” Participants also enjoyed being exposed to the benefits of having student analyze correct and incorrect worked examples as an instructional strategy. Compared with traditional note-taking, analyzing correct and incorrect worked examples promotes mathematical problem-solving and reasoning and leads to a conceptual understanding of mathematics (Booth et al., 2013; Booth et al., 2014; Star et al., 2015). "Error analysis in general is a nice education tool," responded participant 2 who enjoyed "the final activity with incorrect and correct worked examples" the most. Finally, participant 7 enjoyed "the activities where we were provided with resources that we could select from and then plan realistic, feasible, beneficial activities that would implement those resources." The value that participants placed on learning context specific strategies may explain the high mean score (4.8; mean range 1 – 5; $N = 9$) on the learned strategy item on the Panorama survey.

Qualitative data suggest that active engagement opportunities contributed to participants' positive perception of the SAPD program. Participants described the SAPD program as "more personal" (Participant 5) compared with prior professional development opportunities and stated that the learner-centered designed was "very helpful in developing [participants'] understanding of, not only what reform-minded instruction (entails), but the way it can be implemented in classrooms" (Participant 4). Almost half of the participants (44.4%; N = 9) identified active engagement as contributing the most to their understanding of reform-minded instruction. Participant 5 appreciated "the active engagement and being able to try the strategies" and participant 3 preferred experiencing the strategies "as if (we) were the students." Participants' perceptions of the learner-centered approach can be summarized by the following observation: "we were active participants. We just didn't listen to a presentation. We had ownership in the professional development" (Participant 9).

Consistent with the extant literature on teacher professional development (Darling-Hammond et al., 2017; Desimone & Hill, 2017), participants identified collaborative opportunities as a meaninful component of the SAPD program. "Being able to discuss things and actually participate with your coworkers on meaningful, everyday things" (Participant 1) was regarded as a contributor to changes in knowledge and implementation of reform-minded instruction. The designation of collaboration as meaninful by 44.4% of the participants may explain why participants rated colleagues' ideas as quite helpful on Panorama survey item three.

Several participants described collaborative activities as contributing to personal growth, supporting the mean rating of 4.8 on item one of the Panorama survey. Participant 1 states that "learning from my neighbor is where I have always grown the most." Expressing a desire to continue personal growth, participant 2 remarked "we need to continue to inspire each other to keep the momentum going." Participant 3 noted "I gained more [from the SAPD program] than when they send us to other professional development" and attributed this to having the opportunity to work with co-teaching partners and coworkers who taught the same content. Overall, participants stated that "collaboration was beneficial" (Participant 7).

As indicated by a mean score of 4.8 (score range 0 – 5; $N = 9$) on Panorama survey item one, participants found the SAPD program to be supportive of professional growth. Facilitator support strengthens learning outcomes of professional development programs (Jensen et al., 2016; Learning Forward, 2011) and provides support for participants' changes in beliefs and implementation of reform-minded instruction. Qualitative data provide insight into specific SAPD program components that participants felt supported their professional growth.

Participants cited individual classroom modeling and coaching sessions as SAPD program components that supported instructional development. Participant 6 describes how modeling led to professional growth: "I liked the modeling because I could see how the students reacted and what was happening during instruction. This allowed be to be able to plan how to adjust and improve [my teaching] for the next lesson." Additionally, participant 8 found that "the modeling helped the most because it showed me how to implement the strategy" in the classroom. Participant 8 also stated that adding "more modeling sessions" would be an improvement to the program. As the SAPD program progressed, modeling sessions transformed into coaching sessions for some participants. Participants 1 and 3 indicated that they enjoyed coaching sessions the most out of all SAPD program activities and saw coaching sessions as an opportunity to team teach. Additionally, participant 3 noted that coaching sessions were unique to the SAPD program; "other professional development just gives you a strategy and expects you to teach it. Especially in math, it [classroom modeling] helps to see how [the strategy] works in the actual classroom."

Participants found that the "small and intimate" (Participant 8) context of the SAPD program provided an opportunity for individuals to "ask questions and have things explained" (Participant 8). Individual feedback was valued by participant 2 who expressed that the program "was a lot closer to home in terms of getting feedback that was directly related to instruction." Participant 2 also indicated that the program provided a chance for self-reflection and stated that "it let me step back and look at how I teach and the way kids learn." Similarly, participant 4 expressed that the SAPD program "helped [me] identify that areas I was weak in." Qualitative data illustrate that elements of support were received by participants as intended.

Along with knowledge and pedagogical beliefs, the SAPD program intended to change participants' instructional practices. Professional development that targets instructional change requires time (Desimone, 2009). Qualitative data suggest that participants valued the duration of the program. Participant 1 describes that importance of time to the success of professional development; "I think the duration of the training is really important. When [professional development] is just given one time, I don't think it does any good at all. Having the focus for the entire year really makes a big difference." Participants 2, 6, and 7, agree that the year-long duration contributed to the effectiveness of the SAPD program and also expressed a desire to continue the program focus into the proceeding school year. Participant 6 stated "I would love if there was another [program]" and participant 7 desired "a follow-up that is based on new research findings in order to keep the growth moving forward." Participant 2 stated that continued development would lead to sustained implementation: "I hope we can continue this discussion in the future. I don't want to fall into [my] old habits of only lecture." Notably, participant 2 added "please continue this in the future" on the Panorama closed-ended survey. The desire for continuation of the SAPD program reflects the value participants found in the program and supports outcome evaluation findings.

The theme of leadership emerged from participant responses to semi-structured interview items. Professional development programs that yield high learning outcomes for participants view learning as a responsibility shared between facilitators and participants (Jensen et al., 2016; Learning Forward, 2011). Participant responses indicate that learning was a shared endeavor. Participants felt "valued by the presenter" (Participant 7) and appreciated having "ownership in the professional development" (Participant 9). Participant 4 felt that the SAPD program was effective and attributed it to leadership style: "I think a lot of it had to do with who was doing the professional development. If someone did not have the understanding [the facilitator did] it would not be as effective" (Participant 9). When discussing potential changes to future programs, participant 6 expressed concerns regarding leadership stating: "My only concern in doing this again would be [with] who leads the program. Not all facilitators would be as helpful." Positive perceptions of program leadership support a mean score of 4.8 (mean range 1 – 5; N = 9) on the Panorama survey item regarding facilitator support.

Least meaningful (RQ5). When describing SAPD program components that were least meaningful to changes in reform-minded knowledge, beliefs, and instructional practices, most participants (77.8%; $N = 9$) indicated that all program elements played a factor in their professional growth. The constructive responses by two participants, along with all participants' responses to additional survey questions, provides insight into program elements that participants (a) considered barriers to professional growth and (b) would change for future implementation. Cohering to earlier analyses, the analysis of participant responses is organized using SAPD program foci as themes.

Quantitative ratings on the Panorama survey (Panorama Education, 2015) suggest that participants viewed the SAPD program content as quite relevant to the content they teach. Although the mean rating for this survey item was 4.2 (mean range 1 – 5; $N = 9$), this survey item received the second lowest rating among all survey items. Participants' responses during semi-structured interviews reveal the desire for future professional development to be further situated by course (e.g., Algebra, Geometry, Calculus). Participant 2 expressed the desire for professional development to be "more subject specific." "If I had to come up with some change, I would make it more specific to individual course, [such as] algebra, geometry, etc.…" Participant 6 agreed and also stated that "some of the higher ideas did not seem to directly apply to my students." Participants expressed a desire for an extension of the program to include information on "the type of accommodation [that] would be necessary to implement different types of reform-minded instruction" (Participant 7) for students with learning disabilities.

Results of the Panorama survey indicate that participants found colleagues' ideas quite helpful to professional growth. Qualitative data reveal that participant collaboration was sometimes a barrier to instructional development.

> The hardest thing for somebody like me, that is a change adopter, is trying to relate to coworkers who are resistant to change. A road block in professional development in general is dealing with the frustration of having peers that will resist change just for the sake of resisting. (Participant 1)

Other participants also described divergent thinking as a road block to professional growth. "I felt that some of the discussions were not beneficial, not because of the facilitator, but because of difference in the way other participants view the discussion" (Participant 7). The SAPD program was designed to encourage collaboration among teacher-based team members, but as expressed by participant 6, "if the groups were consistent with like-minded teachers [rather than like-subject], I would have preferred that." Not only does the view of divergent thinking as less productive challenges the premise of reform-minded instruction (Sawada et al., 2002), but without challenging personal beliefs and habits of mind (Mezirow, 1997), professional growth could have been inhibited.

Qualitative data suggest that participants viewed time as a constraint within the SAPD program. Logistics required that the SAPD program sessions be conducted during afternoon departmental and TBT meeting times. Some participants found this constraining. "The time of the program, dictated by TBT time, was what I enjoyed the least. Having more choice of when to meet would have been better" (Participant 6). Participant 5 concurred stating, "I would like the presentation time to be more flexible." Mandated meeting time could also impact participant responsiveness to the SAPD program as discussed in the preceding section.

Participants also viewed time as a constraint to implementing reform-minded instructional practices. The recognition the "this type of teaching requires in depth planning and execution" (Participant 5) and that "you need more time to come up with the lessons" (Participant 2) is a barrier to change. Illustratively:

> To do things right does take more prep time, before the fact. The [question] you have to deal with is 'is this the easiest thing for me to do or does it take prep time to make it happen?' This is the biggest road block to change, not just with this program, but with any professional development. (Participant 1)

Participant 3 also recognized that professional growth requires a time commitment stating, "I would make the meetings mandatory or you would have to make it up if you missed [a session]. You have to be present at all meetings to gain the most."

Fidelity of Implementation

In this section, I examine the degree to which the SAPD program was implemented as designed (RQ6). The ability of the SAPD program to induce changes in reform-minded instructional knowledge, beliefs, and practices is dependent on implementation fidelity including measures of dose delivered, dose received, program adherence, and participant responsiveness (Dusenbury et al., 2003). This examination of implementation fidelity includes a quantitative analysis of delivery and attendance logs and a qualitative analysis of SAPD session recordings and post-intervention semi-structured interviews.

The SAPD program intended to deliver 15, one hour face-to-face instructional sessions and six full period (45 minutes) classroom modeling sessions. Quantitative data from researcher records indicate that this goal was not met. Rather, the SAPD program actually delivered 11 face-to-face instructional sessions and five classroom modeling sessions. Participant attendance rates at face-to-face and instructional modeling sessions are included in Table 5.1

Table 5. 16

Summary of Participation Rates by SAPD Program Activity

SAPD Program Activity	Number of Participants in Attendance (*N* = 9)	Participation Rate
Session 1	8	88.9%
Session 2	7	77.8%
Session 3	8	88.9%
Session 4	7	77.8%
Session 5	7	77.8%
Session 6	8	88.9%
Session 7	8	88.9%
Session 8	5	55.6%
Session 9	8	88.9%
Session 10	5	55.6%
Session 11	8	88.9%
Modeling 1	9	100%
Modeling 2	8	88.9%
Modeling 3	8	88.9%
Modeling 4	8	88.9%
Modeling 5	9	100%
Participants completing all program activities	5	55.6%

Participant rates for classroom modeling session were generally higher than participant rates for face-to-face sessions. The average participation rate from classroom modeling sessions was 93.3% (N = 9). Two classroom modeling sessions achieved 100% (N = 9) participation and the remaining three sessions reached 88.9% (N = 9) participation rates. The average participation rates for face-to-face sessions 72.7% (N = 9). While most face-to-face sessions (6 sessions) achieved an 88.9% (N = 9) participation rate, no session reached 100% attendance and two sessions resulted in a 55.6% (N = 9) participation rate. Quantitative data indicate that only five participants were in attendance at all SAPD sessions. Delivery and participation rates provide quantitative measures of program fidelity. Program delivery and participation rates were impacted by the decision to implement the program during designated meeting time. Delivery was impacted by administrative decisions to alter or cancel scheduled meetings. Participation rates at delivered sessions were impacted by conflicting schedules or participants' absence from school on meeting dates. Due to the collaborative and interactive nature of the SAPD program, participants were unable to make up sessions due to absences. Dosage, both delivered and received, is only one measure of implementation fidelity (Dusenbury et al., 2003). Adherence to program objectives and participant responsiveness are additional measures of fidelity (Dusenbury et al., 2003) and can be explained by a qualitative analysis of SAPD recordings and post-intervention semi-structured interviews.

Program Adherence

Research informed professional learning designs reflect a shift from off-site, transmission models to embedded, interactive models (Avalos, 2011) where active engagement is prioritized (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Garet et al., 2001). Instructional coaching and modeling provide opportunities for experimentation and reflection, crucial elements in educator professional development (Clarke & Hollingsworth, 2002). Finally, opportunities for collaboration elevate the expertise of all participants (Desimone & Hill, 2017). As described in Chapter 4, the SAPD program was designed to immerse participants in active learning experiences situated in participants' professional context. The SAPD program was designed to be responsive to participants' needs and support professional growth through classroom modeling and opportunities for peer collaboration. A qualitative analysis of adherence to program objectives follows.

Qualitative data provide an explanation for successful program outcomes given low rates of dose delivered and dose received. Data from SAPD session recordings indicate that the SAPD program was responsive to participants' learning needs, a differentiating factor between effective and ineffective professional development (Fisher & Frey, 2014). While the initial SAPD session introduced participants to the characteristics of reform-minded instruction and outlined SAPD program objectives, subsequent sessions were planned in response to participants' questions, interests, challenges, classroom needs, and course content.

SAPD session recordings provide evidence that face-to-face sessions were responsive to participant needs. The responsive design is illustrated by facilitator's remarks:

> What I have heard, over the last few sessions, is that some of these strategies take a lot of time. (Instructional time) is a concern for us. Today's session might help address that concern.

The facilitator also involved participants in decisions regarding SAPD content. The SAPD program involved an in-depth study of three reform-minded instructional strategies, voted on by participants. Participants completed a Google Form to indicate personal preferences of strategies.

An analysis of semi-structured interview responses illustrate that participants found the SAPD content "directly related to (mathematics) instruction" (Participant 2) compared with professional development that covers a broad spectrum of disciplines. Participant 6 described the program as "applicable to my exact field and discipline and what I do in my classroom."

Participant 3 would have preferred “an even narrower focus” with more “one-on-one instruction.”

Researchers concur that active engagement is critical to the success of professional development programs (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Garet et al., 2001). Active engagement occurs when teachers observe, reflect, discuss, and construct knowledge through interactions with the content and with colleagues (Learning Forward, 2011). Pedagogical expertise stems from these critical interactions (Bryk, 2017), a premise indicative of a sociocultural perspective of learning (Raphael et al., 2014). Clarke and Hollingsworth (2002) and Darling-Hammond et al. (2017) concur that learner-centered approaches to professional development lead to instructional change.

Just as mathematics reform efforts emphasize student-centered instruction for optimal mathematics achievement, the SAPD sessions were designed to engage participants as learners in reform-minded experiences. Each of the fifteen SAPD sessions included an opportunity for participants to experience a reform-minded lesson through the eyes of a learner. The following excerpt from an SAPD session concentrating on student exploration exemplifies the interactive and experiential nature of the SAPD sessions. In this session, participants were introduced to 3 act tasks, strategies to promote student exploration and mathematical modeling. The opening of the 3-act task required participants to *wonder* about the image in Figure 5.3.

Figure 5. 3. Image from Donut Delight 3-act math task (Pearce, 2018)

Facilitator:	When you see this picture, what do you wonder?
Participant 2	I think that is a fake picture.
Participant 10	You're right, that would be a heavy box.
Participant 2	What do Krispy Kreme donuts taste like?
Participant 10	How many donuts?
Participant 3	How much would that weight?
Participant 5	It's a waste of sugar!
Participant 1	Do we get to taste one?
Participant 2	You can't get our hopes up with all these images.
Facilitator	What else do you wonder that is specifically mathematical?
Participant 2	Would it sink in the middle like bookshelves?
Participant 10	How much does a donut weigh?

Participant 2 How many people would you need to carry that?

Participant 1 How much of the box is wasted space?

Participant 2 How fast could you eat those donuts?

Participant 3 Donuts per second…

Participant 4 What does the machine look like that cuts the donut?

Participant 2 How does the glaze get on the donut?

Facilitator: Make a prediction. How many donuts are in that box?

Participant 10 They are just painted on the cover.

Participant 6 They look like cheerios.

Participant 2 They are not donuts.

Participant 10 216.

Participant 2 You counted rows.

Participant 10 It's more than 30.

Participant 6 501.

Participants' discourse in this excerpt illustrates the active role of participants in the learning process. SAPD active learning experiences, such as the 3-act task experience, illustrated in above were followed with (a) a discussion of the reform-minded characteristics included in each activity, (b) collaboration with teacher-based teams (TBT) to design a lesson using the suggested strategy, and (c) group sharing of reflections and anticipated challenges.

Consistent with a sociocultural perspective of learning, the SAPD program emphasized collaboration as a means from professional growth. Evidence that the SAPD program adhered to a collaborative design exists in participant responses to semi-structured interviews. The majority of participants (66.7%; N = 9) mentioned collaboration when describing program elements that were either enjoyed the most or contributed to changes in knowledge and implementation of reform-minded instruction. SAPD session recordings indicate that participants collaborated on knowledge building activities and planning lessons during all ($N = 11$) SAPD sessions. SAPD sessions artifacts (see Figure 5.4) provide evidence of participants' collaborative efforts during SAPD sessions.

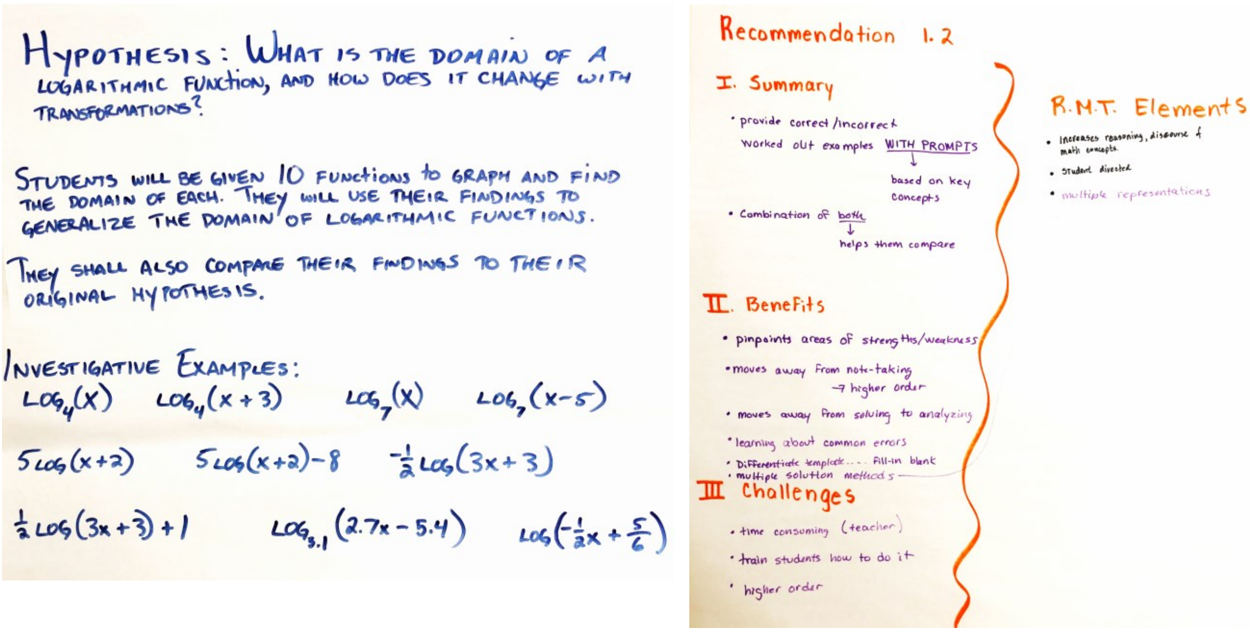

Figure 5. 4. Illustrations of TBT collaborative work during SAPD session activities.

Participants collaborated on subject specific lessons involving the use of creating and testing hypotheses. The collaborative efforts of the pre-calculus TBT are illustrated on the left in Figure 5.4. Participants also collaborated on knowledge building activities. For example, participants were asked to consider the benefits and challenges to using correct and incorrect worked examples as an instructional tool. One team's collaborative efforts are illustrated on the right in in Figure 5.4.

The construct of support underscores a sociocultural perspective of professional learning (Gavelek & Raphael, 1996; Raphael et al., 2014). Modeling and coaching targets teachers' zone of proximal development (Vygotsky, 1978) along with increasing the likelihood of sustained implementation of learned strategies (Jensen et al., 2016; Learning Forward, 2011). Quantitative findings indicate that that the SAPD program failed to meet the program goal of offering six instructional modeling sessions. However, a mean score of 4.8 (mean range 1 – 5; N = 9) on the Panorama survey item related to facilitator support indicates that the SAPD program adhered to its goal to support participants in understanding and implementing reform-minded instruction.

Qualitative data support participants' rating of the SAPD program as quite supportive and provide further evidence of adherence to program objectives. During post-intervention semi-structured interviews, participants differentiated the SAPD program from prior professional development experiences responding that the SAPD program provided feedback (Participant 2), included one-on-one coaching (Participant 3), and provided opportunities for questioning and clarification (Participant 8). Modeling and coaching sessions were listed among the SAPD program activities that participants enjoyed the most:

> I liked the modeling because I could see how the students reacted and what was happening with the during the instruction. This allowed me to be able to plan how we could adjust (and) improve for the next lesson (Participant 6).

In addition to classroom modeling sessions, the SAPD program intended to encourage collegial support. Participant 6 describes being supported by colleagues stating, "being in the program and having time to collaborate with other intervention specialists supported my learning." Similarly, participant 1 preferred "learning from my neighbor" and states that "if there was a change to move forward with the program, being able to observe each other within the department would be neat."

Participant responsiveness

Participant responsiveness, the degree to which participants are engaged in and transformed by the program activities, contributes to implementation fidelity (Dusenbury et al., 2003). Active participation is critical to effective PD (Polly & Hannafin, 2011) especially when a change in instruction is involved (Guskey, 1986). Participants' attendance at SAPD program sessions predicates participant responsiveness and, as indicated by participation rates was inconsistent. In fact, only 55.6% ($N = 9$) of participants attended all program sessions. The relative success of the SAPD program can thus be attributed to participant responsiveness to program activities.

SAPD session recordings provide evidence that participants fully engaged in program activities. During SAPD face-to-face sessions, participants experienced reform-minded instructional strategies from the perspective of a student. The following audible excerpt from a face-to-face session illustrates high-levels of engagement by participants:

Participant 3 This hurts my head.

Participant 2 Cognitive dissonance, the sounds are distracting me…

Participant 1 I'll turn it up.

Participant 2 This one is terrible.

Participant 10 I don't know what I am looking for.

Participant 2 Can I get actually ice cream here? I got far…

Participant 9 I don't' even know how to do it.

Participant 3 Oh, it's not what it is.

Participant 3 Now you can buy it because you have 70 cents.

Participant 9 I like this.

Participant 3	It hurts my head, I feel like I should be better. Can I have a calculator?
Participant 10	It took me awhile to understand.
Participant 9	I thought that was 7 cents.
Participant 2	It's not what it's supposed to be.
Participant 1	What do you mean?
Participant 2	I think I did it.
Participant 3	I can't buy milk.
Participant 1	I got to the pretzels.
Participant 9	Shoot.
Participant 2	Yeah! I got the milk.
Participant 3	You cheated.
Participant 2	How did I cheat?
Participant 1	What products did you get?
Participant 2	Maybe if you followed my strategy…
Facilitator	What was your strategy?
Participant 2	I showed….
Participant 10	If you get to a certain point..
Participant 3	When you have 10 seconds left..

This interaction exemplifies the responsiveness of participants to all program activities. These data suggest that the SAPD program was not only implemented as designed, but was positively received by the engaged participants.

Limitations

There are several limitations to this research impacting the conclusiveness of the study. These limitations concern the sample size and composition, absence of a control group, the timing of SAPD face-to-face sessions, and biases due to the collegial relationships among the participants and the researcher. The limited sample size for this study ($N = 9$) is a threat to statistical conclusion validity. The Wilcoxon Signed Ranks Test detected a significant difference between participants' beliefs and use of reform-minded instructional practices, yet the small sample size used in this study compromises the validity of these conclusions. Although steps were taken to strengthen statistical conclusion validity, such as using reliable instrumentation to measure study outcomes (Shadish et al., 2012), the issue of sample size remains a concern.

Participants for this study were chosen through convenience sampling which, together with sample size concerns, impacts the ability to generalize study findings beyond this population of mathematics teachers. Eligibility criteria and logistical constraints resulted in a sample representing 75% ($N = 12$) of teachers in the mathematics department of this high-school. As such, participants were co-workers who interacted on a daily basis. Although generalizability remains a concern, successful outcomes can be attributed to the familiarity of participants with one another and the establishment of a culture of learning (Jensen et al., 2016) within the mathematics department. As qualitative findings suggest, participants valued the support and collaboration offered by the SAPD program.

The absence of a control group also impacts the strength of research findings (Shadish et al., 2002). In fact, consideration was given to implementing an untreated control group design (Shadish et al., 2002) with teachers from the second high school in the district serving as a control group. The benefits of adding the control group were weighed against increased threats to validity due to group differences (Shadish et al., 2001). Validity concerns, logistic concerns, and limited researcher resources made the one-group pretest-posttest design the most viable option for this study, despite weakened statistical strength.

SAPD face-to-face sessions were scheduled in place of departmental and TBT meetings. Departmental and TBT meeting times are during by school administrators prior to the beginning of the school year. Although participants consented to meet during contractual time, the scheduling of SAPD session during this time impacted the fidelity of implementation. Administration's decision to cancel departmental and TBT meetings and participants' absence from school on meeting days affected measures of dose delivered and dose received. Furthermore, participant attendance at SAPD sessions to fulfill contractual obligations cannot ensure active participation or responsiveness, critical components of program fidelity (Linnan & Steckler, 2002). However, qualitative data from SAPD session recordings and artifacts indicate that participants were engaged and responsive to program activities.

The collegial relationship between the researcher and participants introduces response bias to the study. The SAPD program was designed, facilitated, and evaluated by a single researcher who is also a teacher in the mathematics department at this high school. It is likely that participants provided socially acceptable or desirable responses during SAPD sessions, on SAPD artifacts and instrumentation, and during semi-structured interviews. Furthermore, the desire to assist a colleague could have been a motivating factor for participants' willingness to partake in the study and work toward program objectives.

High-performing educational systems recognize the value of collegial leadership to educator growth (Calvert, 2016; Jensen et al., 2016; Learning Forward, 2011). Bias created by the collegial relationship between the researcher and the participants may be unfounded given the impact that teacher agency has on professional development and growth (Calvert, 2016).

Teacher agency involves the recognition that collegial collaboration promotes professional growth (Calvert, 2106), aptly described by participant 2 who stated, "we need to continue to inspire each other." Leadership of the SAPD program was shared as the researcher and participants engaged in program components as a learning community.

Discussion and Implications

This research bears important implications for the educator professional development research community. Professional learning designs have evolved from one-shot workshop models (Van Broekhuizen & Dougherty, 1999) to models reliant on interactions among participants, peers, and practice (Clarke & Hollingsworth, 2002). In fact, prior to 2001, researchers understand little about how professional development activities translated into educator growth (Garet et al., 2001). Current literature regards active learning opportunities and participant engagement as essential to improving professional learning outcomes (Darling-Hammond et al., 2017; Desimone & Garet, 2015; Garet et al., 2001).

Implications for Practice

The positive outcomes of this study substantiate the importance of implementing reform-minded professional development as an approach to increasing reform-minded mathematics instruction. Just as mathematics reform efforts emphasize student-centered instruction for optimal mathematics achievement, researchers recommend learner-centered professional development to transform instructional practices (Darling-Hammond et al, 2017; Garet et al., 2001). Participants in this study experienced significant changes (p = .017) in their beliefs regarding reform-minded instruction and significantly (p = .028) increased their use reform-minded instructional practices. Despite threats to statistical conclusion validity, qualitative data suggest that SAPD program design elements; situated, learner-centered, collaborative, and supportive, enhanced participants' experiences in the program. Positive program outcomes confirm research findings indicating that "reform types of professional development may be more likely than traditional forms to make connections with classroom teaching" (Garet et al., 2001, p. 921).

Transforming instructional practices at this high school required a shift from reliance on teacher-centered learning to a purposeful focus on student-centered learning. Research suggests that teachers can continue to improve their instruction in mathematics by using the RTOP as a formative assessment of reform-minded pedagogy (Amrein-Beardsley & Popp, 2011). Sawada et al. (2002) encourage teachers to use the RTOP to set instructional goals in much the same way they set learning goals for their students. Research shows that teachers value the self-reflective nature of the RTOP and purport that purposeful planning for reform-minded practices can lead to sustained implementation (Amrein-Beardsley & Popp, 2001).

High-performing educational systems recruit professional development leaders from among the teaching staff (Jensen et al., 2016). Although my collegial relationship with the participants may have introduced bias, this relationship provided a unique opportunity to examine participant experiences. As a participant observer (Malinowski, 1961) in the SAPD program, I was able to understand participants' perspective, provide responsive support and feedback, and share in the responsibility of improving mathematics instruction (Learning Forward, 2011). Establishing this sense of collective responsibility is crucial to sociocultural theory where learners facilitate one another's growth (Vygotsky, 1978). Participants' positive perception of SAPD program leadership provides insight the characteristics that teachers desire in professional development facilitators.

Qualitative findings suggest that teachers desire professional development programs that provide one-on-one support and allow for collaboration among colleagues with common goals. This individual experience is compromised in programs with a broader scope in terms of content and number of participants (Garet et al., 2001). Individualized professional development requires an investment of time and financial resources, but as seen with higher performing systems, contributes to a system-wide culture of learning (Jensen et al., 2016).

Implications for Research

Although the small sample size compromises this study's power to detect a causal relationship, the SAPD program can serve as a pilot study to inform future mathematics professional development programs. Similar to an evaluability assessment (Strosberg & Wholey, 1983) the results of this study can provide insight into the generalizability of the SAPD program and the effectiveness of program components.

Decisions regarding future programing efforts include (a) determining way to increase fidelity of implementation, (b) increasing the sample size to improve statistical conclusion validity while maintaining program levels of collective responsibility, and (c) designing the study to include a control group thereby increasing the validity of the results. However, an expansion of the SAPD program necessitates consideration of the SAPD program elements participants found to be meaningful. Participants' attributed program success to the support and responsiveness embedded in the collaboration and instructional modeling components. Aligning with standards for effective professional development (Jensen et al, 2016; Learning Forward, 2011), the SAPD program endeavored to create a culture of reform-mindedness among program participants. Professional development that consider individual teachers' needs, provide one-on-one support, and remain situated context, advance the effectiveness of the program (Darling-

Hammond et al., 2017). Future research implications include finding ways to balance validity and generalizability while retaining the ability to personalize programming.

Effective educational systems recognize the importance of education professional development as a means to improving student learning (Jensen et al., 2016; Learning Forward, 2011). Although models of professional learning differ in terms of where they place student outcomes in their respective change processes (Desimone, 2009; Guskey, 2002; Clarke & Hollingsworth, 2002), all view improved student outcomes as the primary goal of professional development. Future research implications include replicating study to incorporate measurements of student mathematics achievement.

Conclusion

This research study examined the knowledge, beliefs, and implementation of reform-minded instruction of a cohort of nine high-school mathematics teachers. Included in this study was an investigation of the effectiveness of a situated apprenticeship professional development program at inducing change in these dependent variables. As a result of participating in the SAPD program a significant change in participants' beliefs regarding reform-minded mathematic instruction occurred. Additionally, participants' implementation of reform-minded instructional practices significantly increased. These findings are perhaps even more remarkable considering program delivery and participation rates were less than anticipated. The relative success of the SAPD program in spurring changes in participants' beliefs and instructional practices can be attributed to high levels of participant responsiveness, strong adherence to program goals, and the research-informed program design.

In theory, professional development programs that implement the standards recommended by research (Darling-Hammond et al., 2017; Garet et al., 2001) and Learning

Forward (2011) demonstrate a sociocultural perspective of learning. Along with developing expertise, learning that occurs in a social context increases teacher agency and motivation (Calvert, 2016; Clarke & Hollingsworth, 2002) contributing to a sense of collective responsibility (Jensen et al., 2016). Teacher agency, an individual's capacity to initiate personal and collegial growth (Calvert, 2016), exemplifies learning as a social construct (Vygotsky, 1978) and forms the foundation for the development of a system-wide culture of learning (Jensen et al., 2016; Learning Forward, 2011). Applied to professional development, sociocultural approaches to learning capitalize on the expertise of individual participants in support of collective improvement (Rohlwing & Spelman, 2014).

The conceptual framework guiding this study (Figure 1.2) recognized the contributions of mathematics content knowledge, pedagogical content knowledge, teacher expectations, and teacher beliefs to mathematics educators' choice of instructional practices. Derived from an ecological systems perspective (Bronfenbrenner, 1996), the framework recognizes that pedagogical decisions stem from interactions between complex factors. An additional factor contributing to educators' choice of instructional practices emerged from this study. Qualitative data reveal that measures of accountability, namely high-stakes testing, influenced pedagogical methods. Participants' concerns align with current research suggesting that preparations for high-stakes exams influence instructional practices (Au, 2007; Gebril & Eid, 2017). An altered conceptual framework would include factors of accountability.

Professional development aims to advance student learning through a focus on teacher learning (Learning Forward, 2011). Responding to calls for reform in mathematics instruction (NCTM, 2014; National Mathematics Advisory Panel, 2008) originates in reforming educator professional development. Learner-centered professional development begets learner-centered mathematics instruction (Polly & Hannafin, 2010), proven to increase student outcomes in mathematics (Jong et al.,2010; Krupa & Confrey, 2017; Lawson et al., 2012).

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Appendix A

Needs Assessment Teacher Questionnaire

1. Please indicate the level of experience you have in each of the areas below, by circling one choice for each item.

    a. How many years of teaching experience do you have?

    A. 0-1 years B. 2-5 years C. 6-10 years D. More than 10 years

    b. How many years of teaching experience do you have in this school district?

    A. 0-1 years B. 2-5 years C. 6-10 years D. More than 10 years

    c. How many years of experience do you have teaching algebra?

    A. 0-1 years B. 2-5 years C. 6-10 years D. More than 10 years

    d. How many years of experience do you have teaching students with disabilities?

    A. 0-1 years B. 2-5 years C. 6-10 years D. More than 10 years

2. Please check the appropriate line, indicating the highest level of formal education you have completed.

    ______ Bachelor's Degree
    ______ Master's Degree
    ______ Master's Degree +15
    ______ Master's Degree +30
    ______ Master's Degree +45
    ______ Doctoral Degree

3. Please indicate how strongly you agree with each of the following statements about your beliefs about teaching and learning, by circling one choice for each item.

    a. My role as a teacher is to facilitate students' own inquiry

    A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

    b. Students learn best by finding solutions to problems on their own

    A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

    c. Students should be allowed to think of solutions to problems themselves before the teacher shows them how they are solved

    A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

d. Thinking and reasoning processes are more important than specific curriculum content.

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

e. Students learn best when they are in classrooms with classmates of similar ability

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

f. Algebra concepts should be taught before skills are developed

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

g. Procedural skills should be introduced by the teacher and then practiced by the students

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

4. Please indicate how strongly you agree with each of the following statements regarding the Ohio Learning Standards, by marking one box for each statement.

a. I am knowledgeable of the Ohio Learning Standards for algebra

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

b. I consult the Ohio Learning Standards for Mathematics as I plan my lessons

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

c. I would like to have a better understanding of the exact learning standards that comprise the algebra curriculum

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

d. I would like more professional development on understanding the Ohio Learning Standards for algebra

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

e. I would like more professional development on how to implement the Ohio Learning Standards for algebra

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

f. I would like more professional development on how to implement the Mathematical Practices of the Ohio Learning Standards for Mathematics

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

5. Please indicate how often each of the following occurs in your algebra class, by circling one choice for each item.

   a. I assess students' prior knowledge, and clarify misconceptions before introducing new content

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   b. Students work in small groups to explore a topic before I provide instruction

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   c. Students work in groups based on ability

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   d. I give different work to the students who have difficulties learning

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   e. I give different work to the students who can advance faster

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   f. I refer to a problem from everyday life or work to demonstrate how new knowledge is useful

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   g. I present material to the whole class through lecture or working out examples

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   h. I work through examples, explaining algorithms and procedures

      B. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   i. I allow students time to explore alternative methods of problem solving

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   j. Students work on projects that require at least one week to complete

      A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

   k. Students reflect on their learning

A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

l. Students use manipulatives to discover or explain concepts

A. Less than once a week B. 1-2 times per week C. 3-4 times per week D. Daily

6. Please indicate how strongly you agree with each of the following items regarding assessments, by circling one choice for each item.

a. I create my own assessments for each unit of my algebra course

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

b. I use performance tasks to assess student learning at least once a month.

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

c. The tests that I administer contain equal amount of closed-ended and open ended questions

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

d. The tests that I administer contain mostly multiple choice questions

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

e. I provide written feedback on student work in addition to a letter grade

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

f. I examine incorrect solutions on assessments to determine the strengths and weakness of students' procedural and conceptual knowledge

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

g. Students in my class evaluate their own progress

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

h. Students in my class meet with me one-on-one to obtain feedback

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

i. My daily lessons involve providing individual feedback to students as they work on tasks

A. Strongly Disagree B. Disagree C. Agree D. Strongly Agree

Appendix B

Reformed Teaching Observation Protocol (RTOP)

Daiyo Sawada — External Evaluator
Michael Piburn — Internal Evaluator

and

Kathleen Falconer, Jeff Turley, Russell Benford and Irene Bloom
Evaluation Facilitation Group (EFG)

Technical Report No. IN00-1
Arizona Collaborative for Excellence in the Preparation of Teachers
Arizona State University

I. BACKGROUND INFORMATION

Name of teacher ______________________ Announced Observation? ______________________
(yes, no, or explain)

Location of class ____
(district, school, room)

Years of Teaching ____________________ Teaching Certification ____________________
(K-8 or 7-12)

Subject observed ______________________ Grade level ______________________________

Observer ____________________________ Date of observation ________________________

Start time ___________________________ End time _________________________________

II. CONTEXTUAL BACKGROUND AND ACTIVITIES

In the space provided below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity) and teacher that you think are important. Use diagrams if they seem appropriate.

Record here events which may help in documenting the ratings.

Time	Description of Events

III. LESSON DESIGN AND IMPLEMENTATION

		Never Occurred				Very Descriptive
1)	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	0	1	2	3	4
2)	The lesson was designed to engage students as members of a learning community.	0	1	2	3	4
3)	In this lesson, student exploration preceded formal presentation.	0	1	2	3	4
4)	This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.	0	1	2	3	4
5)	The focus and direction of the lesson was often determined by ideas originating with students.	0	1	2	3	4

IV. CONTENT

Propositional knowledge

6)	The lesson involved fundamental concepts of the subject.	0	1	2	3	4
7)	The lesson promoted strongly coherent conceptual understanding.	0	1	2	3	4
8)	The teacher had a solid grasp of the subject matter content inherent in the lesson.	0	1	2	3	4
9)	Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.	0	1	2	3	4
10)	Connections with other content disciplines and/or real world phenomena were explored and valued.	0	1	2	3	4

Procedural Knowledge

11)	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	0	1	2	3	4
12)	Students made predictions, estimations and/or hypotheses and devised means for testing them.	0	1	2	3	4
13)	Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.	0	1	2	3	4
14)	Students were reflective about their learning.	0	1	2	3	4
15)	Intellectual rigor, constructive criticism, and the challenging of ideas were valued.	0	1	2	3	4

Continue recording salient events here.

Time	Description of Events

V. CLASSROOM CULTURE

Communicative Interactions

		Never Occurred				Very Descriptive
16)	Students were involved in the communication of their ideas to others using a variety of means and media.	0	1	2	3	4
17)	The teacher's questions triggered divergent modes of thinking.	0	1	2	3	4
18)	There was a high proportion of student talk and a significant amount of it occurred between and among students.	0	1	2	3	4
19)	Student questions and comments often determined the focus and direction of classroom discourse.	0	1	2	3	4
20)	There was a climate of respect for what others had to say.	0	1	2	3	4

Student/Teacher Relationships

21)	Active participation of students was encouraged and valued.	0	1	2	3	4
22)	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	0	1	2	3	4
23)	In general, the teacher was patient with students.	0	1	2	3	4
24)	The teacher acted as a resource person, working to support and enhance student investigations.	0	1	2	3	4
25)	The metaphor "teacher as listener" was very characteristic of this classroom.	0	1	2	3	4

Additional comments you may wish to make about this lesson.

Appendix C

Reform-Minded Teaching
Exit Ticket - SAPD Cycle 1

Take moment to reflect on your current understanding of reform-minded mathematics instruction. For each item below, choose the answer that best describes reform-minded instruction.

Lesson Design and Implementation

I. Reform-minded mathematics instruction begins with:

a. Asking students if they have any questions on prior material
b. The instructor modeling mathematical procedures
c. Student exploration or investigation of concepts
d. A pre-assessment to determine students' prior knowledge

Propositional Pedagogical Knowledge

II. Reform-minded mathematics lessons predominantly focus on

a. mathematical procedures
b. mathematical skills
c. mathematical concepts
d. mathematical rules

Procedural Pedagogical Knowledge

III. In reform-minded mathematics lessons, students learn to

a. predict
b. accept
c. repeat
d. converge

Communicative Interaction

IV. In reform-minded mathematics classrooms, students are

a. passive listeners
b. active learners
c. unilateral thinkers
d. quiet followers

Student-Teacher Relationships

V. Students would describe reform-minded teachers as

a. tolerant
b. patient
c. impersonal
d. all-knowing

VI. What questions or comments do you have at this time regarding reform-minded instruction?

Reform-Minded Teaching
Exit Ticket - SAPD Cycle 2

Take moment to reflect on your current understanding of reform-minded mathematics instruction. For each item below, choose the answer that best describes reform-minded instruction.

Lesson Design and Implementation
I. Reform-minded mathematics instructors:
  a. guide students toward convergent thinking
  b. allow students time to explore prior to explication
  c. design the sequence of the lesson, anticipating possible areas for misconceptions
  d. group students by ability to differentiate their learning experiences

Propositional Pedagogical Knowledge
II. Reform-minded instructors teach abstract concepts through
  a. explicit instruction
  b. multiple representations
  c. guided practice of procedures
  d. worked examples

Procedural Pedagogical Knowledge
III. Reform-minded mathematics instructors
  a. Differentiate learning objectives for individual students
  b. Maintain rigorous expectations for all students
  c. Recognize the importance of simplified tasks for scaffolding learning
  d. De-emphasize higher order skills for learners who struggle with procedures

Communicative Interaction
IV. Reform-minded mathematics instruction recognizes the value of
  a. divergent thinking
  b. convergent thinking
  c. teacher directed discourse
  d. teacher guided explanations

Student-Teacher Relationships
V. In reform-minded mathematics classrooms instructors are
  a. talkers
  b. directors
  c. managers
  d. listeners

VI. What questions or comments do you have at this time regarding reform-minded instruction?

Reform-Minded Teaching
Exit Ticket - SAPD Cycle 3

Take moment to reflect on your current understanding of reform-minded mathematics instruction. For each item below, choose the answer that best describes reform-minded instruction.

Lesson Design and Implementation
I. In reform-minded mathematics classrooms, students can best be described as:
   a. learners
   b. explorers
   c. individuals
   d. listeners

Propositional Pedagogical Knowledge
II. Reform-minded mathematics lessons
   a. follow a logical sequence from start to finish
   b. are open-ended to allow for exploration
   c. are less structured than traditional mathematics lessons
   d. focus on content over context

Procedural Pedagogical Knowledge
III. In reform-minded mathematics lessons, learners
   a. agree
   b. disregard
   c. reflect
   d. follow

Communicative Interactions
IV. In reform-minded mathematics classrooms, students
   a. are guided by teachers in constructing mathematical knowledge
   b. construct meaning independently
   c. participate in student-student discourse to construct knowledge
   d. construct knowledge by modeling teacher behaviors

Student-Teacher Relationships
V. In reform-minded mathematics classrooms teachers
   a. model procedures
   b. serve as a resource
   c. impart knowledge
   d. explicitly instruct

VI. What questions or comments do you have at this time regarding reform-minded instruction?

Reform-Minded Teaching
Exit Ticket - SAPD Cycle 4

Take moment to reflect on your current understanding of reform-minded mathematics instruction. For each item below, choose the answer that best describes reform-minded instruction.

Lesson Design and Implementation
I. In reform-minded mathematics classrooms, students learn
   a. that there are multiple pathways to problem-solving
   b. that there is a best approach to problem-solving
   c. that some ways of problem-solving are more efficient than others
   d. specific strategies for solving specific problems

Propositional Pedagogical Knowledge
II. Reform-minded mathematics instructors
   a. emphasize conceptual understanding
   b. emphasize procedural understanding
   c. emphasize skill development
   d. emphasize concrete representations

Procedural Pedagogical Knowledge
III. Reform-minded mathematics instructors
   a. promote the use of specific manipulatives to help students visualize concepts
   b. highlight algebraic representations for developing procedural knowledge
   c. suggest the use of specific representations to reduce confusion
   d. encourage the use of a variety of representations to convey meaning

Communicative Interactions
IV. In reform-minded mathematics classrooms
   a. there is a sense of individual accomplishment
   b. the focus is on personal achievements
   c. the responsibility for learning is shared
   d. students rely on the teacher for learning key concepts

Student-Teacher Relationships
V. Reform-minded mathematics instructors
   a. encourage spontaneous student responses to worked examples
   b. provide enrichment activities for students who quickly master concepts
   c. use the majority of class time for direct instruction
   d. allow student to generate alternative solutions

VI. What questions or comments do you have at this time regarding reform-minded instruction?

Reform-Minded Teaching
Exit Ticket - SAPD Cycle 5

Take moment to reflect on your current understanding of reform-minded mathematics instruction. For each item below, choose the answer that best describes reform-minded instruction.

Lesson Design and Implementation
I. Reform-minded mathematics instruction places importance on:
  a. memorization
  b. guided practice
  c. isolated skills
  d. student reflection

Propositional Pedagogical Knowledge
II. Reform-minded mathematics instruction
  a. stresses mastery of mathematics before integrating with other disciplines
  b. focuses on mathematics development in isolation of other disciplines
  c. integrates knowledge of mathematics with other disciplines
  d. places value on content-specific instruction

Procedural Pedagogical Knowledge
III. In reform-minded mathematics lessons, students typically do *not*
  a. synthesize
  b. criticize
  c. memorize
  d. internalize

Communicative Interactions
IV. In reform-minded mathematics classrooms
  a. student-student interactions help shape the lesson
  b. student-student interactions are minimal
  c. student-student interactions are used to reinforce learned skills
  d. student-student interactions occur weekly

Student-Instructor Relationships
V. Reform-minded mathematics instructors
  a. provide detailed feedback on summative assessments
  b. devote the majority of class time to working out examples
  c. provide feedback as a source of instruction
  d. scaffold learning by providing students with procedures to follow

VI. What questions or comments do you have at this time regarding reform-minded instruction?

Appendix D

Mathematics Teaching Pedagogical Beliefs and Discourse Instrument
(Lischka & Garner, 2016)

For each question, choose the one choice that most often describes your beliefs.

1. I believe when introducing a new concept, it is most important to teach mathematics lessons that:
    a. Focus on one idea at a time, emphasizing both reasoning and computational accuracy together.
    b. Combine a variety of ideas and their connections using a problem -solving approach.
    c. Combine at least two ideas and the connections between them.
    d. Focus on one idea at a time, emphasizing computational accuracy before reasoning.
2. I believe the most important role of the mathematics teacher is to:
    a. Convey information to students and evaluate student performance.
    b. Explain reasoning for mathematical processes to students, assist students in clarifying their mathematical understanding and assess their mathematical knowledge.
    c. Provide information to students, question them about their knowledge, and seek to understand their thinking.
    d. Pose problems that engage students in exploring mathematical ideas and assess their mathematical understanding.
3. I believe that students learn mathematics best by:
    a. Paying attention to the teacher and practicing problems.
    b. Exploring student-generated mathematical problems found in their environment.
    c. Taking notes during lessons and asking questions when they don't understand.
    d. Participating in mathematical investigations in which the teacher designs the questions.
4. I believe that it is important for mathematical conversations to most often be in the form of:
    a. Teacher and student discussion driven by student inquiry.
    b. Teacher and student discussion with the teacher initiating questions.
    c. The teacher initiating questions to determine whether or not students have correct answers.

d. Students talking with other students while the teacher facilitates questioning.

5. I believe it is important to learn math because it:
    a. Provides structure.
    b. Promotes logical reasoning.
    c. Is beautiful and creative.
    d. Is useful.
6. I believe that mathematics:
    a. Is invented.
    b. Is already all known.
    c. Exists independent of human thought and is discovered.
    d. Is constructed as a product of social interaction.

7. When I prepare lessons I believe it is most important to consider the following:
    a. Activities or investigations that will assist my students in developing their own understanding about the key mathematical ideas.
    b. Opportunities for group activity to be used after I convey key information.
    c. Explanations I want to give in a class discussion along with questions I want to ask students during the lesson.
    d. Key information I want to convey in a lesson along with student practice problems.

8. In order to teach students how to factor quadratic polynomials, I believe it is most important to:
    a. Present students with the procedure for factoring and then have them practice individually factoring polynomials.
    b. Use manipulatives to demonstrate using an area model for factoring polynomials with the whole class and then have students work in groups to practice factoring polynomials.
    c. Provide student groups with manipulatives and facilitate groups in creating a model for factoring.
    d. Present students with the procedure for factoring and then have them work in groups to practice factoring polynomials.

9. I believe mathematics is mostly:
    a. Problem solving.
    b. Proving existing ideas.
    c. Computation and manipulation.
    d. Creating new ideas.

10. I believe mathematics is most like:

a. A lawyer's courtroom argument.
b. A painting.
c. Cooking.
d. A 1000-piece jigsaw puzzle.

11. I believe that the most important source of mathematical ideas in the classroom is:
    a. The teacher and the students.
    b. The curriculum.
    c. The teacher.
    d. The students.
12. I believe:
    a. Learning is a mostly individual process that is aided by discussion with the teacher.
    b. Learning is an individual process accomplished by the learner alone.
    c. Learning is a process of social construction that takes place through discourse with a variety of others.
    d. Learning is a process that is accomplished through discussion with other learners and a teacher.
13. I believe that the body of mathematical knowledge is:
    a. Fixed with interconnecting structures.
    b. Fixed and predictable.
    c. Surprising, expanding and driven by new problems.
    d. Surprising and investigated through solving of existing problems.
14. I believe that:
    a. There are multiple ways to learn a mathematical topic.
    b. There is a best way to learn a mathematical topic but it may be represented in more than one way.
    c. Mathematics is learned through problem-solving in which multiple pathways to solutions are possible.
    d. There is a best way to learn a mathematical topic.
15. I believe that students learn the process of completing the square best by:
    a. Working in groups to complete several completing the square problems and discussing the solutions with the group.
    b. Repeating the steps of completing the square and explaining them to a classmate.
    c. Memorizing the steps of completing the square and practicing them.

d. Working with a group using manipulatives to derive the process and then generalize it.

16. I believe that eliciting students' mathematical thinking in classrooms should be accomplished by:
    a. The teacher asking students to explain why their answer is valid.
    b. The teacher asking questions of students to check to see if students have the correct answers.
    c. Students questioning each other about their reasoning with teacher facilitation.
    d. The teacher asking students to explain how they solved a problem.
17. I believe it is most important to ask questions during classes:
    a. To assess whether or not students are paying attention.
    b. To encourage further student exploration and, if necessary, change direction of a lesson.
    c. To evaluate student knowledge.
    d. To better understand my students' thinking.
18. I believe it is most important for students to learn to:
    a. Generate and explore their own mathematical questions.
    b. Explain reasoning for processes and explore connections between problems.
    c. Solve problems and explain reasoning for processes.
    d. Solve specific problems accurately.
19. I believe it is most important during lessons to:
    a. Allow students to present solutions only after I have checked them for correctness.
    b. Allow students to present different methods of a solution than I have presented.
    c. Allow students to present solutions and use any misconceptions that surface to propel instruction.
    d. Have the teacher present all solutions so that students are not confused by multiple or incorrect solutions.

20. In order to teach solving linear equations, I believe it is most important to:
    a. Show several examples of solving linear equations with questions asked to check for student understanding incorporated into the demonstration.
    b. Show several examples of solving linear equations and then have students practice solving individually.
    c. Explain the reasoning that creates the rules for solving equations while demonstrating solutions of linear equations.

d. Engage students in conversation that leads to the development of multiple ways to solve linear equations.

Appendix E

Item	Response Anchors				
Overall, how supportive has this professional development provider of your growth as a teacher?	Not at all supportive	Slightly supportive	Somewhat supportive	Quite supportive	Extremely supportive
How valuable was this professional development opportunity?	Not at all valuable	Slightly valuable	Somewhat valuable	Quite valuable	Extremely valuable
How helpful were your colleagues' ideas for improving your teaching during this program?	Not at all helpful	Slightly helpful	Somewhat helpful	Quite helpful	Extremely helpful
How helpful was this professional development opportunity in helping you explore new ideas?	Not at all helpful	Slightly helpful	Somewhat helpful	Quite helpful	Extremely helpful
How relevant has this professional development opportunity been to the content that you teach?	Not at all relevant	Slightly relevant	Somewhat relevant	Quite relevant	Extremely relevant
Through this program, how many new teaching strategies have you learned?	Almost no strategies	A few strategies	Some strategies	Many strategies	A great number of strategies
How much input did you have into individualizing this professional development opportunity?	Almost no input	A little bit of input	Some input	Quite a bit of input	A tremendous amount of input
Overall, how much did you learn about teaching from this professional development?	Learn almost nothing	Learn a little bit	Learn some	Learn quite a bit	Learn a tremendous amount

Panorama Teacher Survey

Appendix F

Mathematics Teacher Interview Protocol

Script

Thank you for taking the time to talk to me today about your experiences as a participant in the SAPD program. I encourage you to be honest in your responses. If you have questions or would like to end the interview at any time, please let me know.

During the interview today, I will be taking type written notes. These notes will be saved on my personal, password protected computer. Prior to saving the notes, I will review them with you to ensure that I have accurately captured your responses. I will ask you for verbal confirmation of the accuracy of my notes.

The responses that you provide will only be shared with my dissertation advisor and dissertation committee members. Your responses may appear in my dissertation work, but your identity will be protected through the assignment of a pseudonym name.

Do you have any questions before we begin?

Interview Questions
***Teachers may be reminded of SAPD components as a source of prompting. Components include (a) choice of instructional strategies, (b) active engagement in instructional strategies, (c) collaboration, (d) classroom modeling, (e) time for reflection, (d) duration, and (e) situated learning.**

1. Compared with professional development you have experienced in the past, how was the SAPD program different?

2. What SAPD program activities did you enjoy the most?

3. What SAPD program activities did you enjoy the least?

4. What aspects or activities of the SAPD program contributed to a change in your instructional practices?

5. What aspects or activities of the SAPD program prevented you from changing your instructional practices?

6. What aspects or activities of the SAPD program contributed the most to your understanding of reform-minded instruction?

7. What aspects or activities of the SAPD program did not impact your understanding of reform-minded instruction?

8. Describe your overall impression of the SAPD program.

9. Describe any changes you would make to the SAPD program if a similar program were to be implemented with other teachers.

10. Is there anything else you would like to share regarding your experiences as a participant in the SAPD program?

Closing

Thank you for agreeing to participate in this interview today and thank you for your valuable feedback. I greatly appreciate your involvement in this project. If you leave this meeting, and think of anything else you would like to add, please do not hesitate to contact me.

Appendix G

Pre-Intervention Survey

1. Please provide your name.

____

2. What is your current employment status as a teacher?

   _____ Full-time

   _____ Part-time

3. How many years of work experience do you have?

   _____ Year(s) working as a teacher at this school

   _____ Year(s) working as a teacher in total

   _____ Year(s) working in other education roles (do not include years working as a teacher)

   _____ Year(s) working in other jobs

4. What is the highest level of formal education you have completed?

_____ Bachelor's Degree	_____ Master's Degree +30
_____ Master's Degree	_____ Master's Degree +45
_____ Master's Degree +15	_____ Doctoral Degree

5. During the past 24 months, did you participate in any of the following professional development activities?

	YES	NO
Knowledge and understanding of my subject field		
Pedagogical competencies in teaching my subject field		

Knowledge of the curriculum

Student evaluation and assessment practices

Information and communication technology skills for teaching
Student behavior and classroom management

School management and administration

Approaches to individualized learning

Teaching students with special needs

Teaching in a multicultural or multilingual setting

Teaching cross-curricular skills (e.g. problem solving, learning-to-learn)

Approaches to developing cross-occupational competencies for future work or future studies
New technologies in the workplace

Student career guidance and counseling

Appendix H

Summary Matrix

Research Questions	Constructs	Measures	Data Analysis
How did mathematics teachers' use of reform-minded instructional practices change as a result of the situated apprenticeship professional development program?	Mathematics teachers use of reform minded instructional practices	Reformed Teaching Observation Protocol (Sawada et al., 2002) SAPD session audio recordings SAPD session artifacts Participant implementation feedback forms Semi-structured interviews	Descriptive Statistics Inferential Statistics A priori coding Emergent coding
How did mathematics teachers' pedagogical beliefs change as a result of the situated apprenticeship professional development focused on reform-minded instruction?	Mathematics teachers' pedagogical beliefs regarding reform-minded instruction	Mathematics Teaching Pedagogical and Discourse Beliefs Instrument (Lischka & Garner, 2016) SAPD session audio recordings Participant implementation feedback forms Semi-structured interviews	Descriptive Statistics Inferential Statistics A priori coding Emergent coding
How did mathematics teachers' knowledge of reform-minded instructional practices change as a result of the situated professional development program?	Mathematics teachers' knowledge of reform-minded instructional practices	Exit tickets SAPD session audio recordings SAPD session artifacts Participant implementation feedback forms Semi-structured interviews	Descriptive Statistics A priori coding Emergent coding

What aspects of the situated apprenticeship professional development do mathematics teachers describe as most impactful in their transformation toward reform-minded instruction?	Mathematics teachers' perception of the situated apprenticeship professional development program	Panorama Teacher Survey (Panorama Education, 2015) Semi-structured interviews	Descriptive Statistics A priori coding Emergent coding
What aspects of the situated apprenticeship professional development do mathematics teachers describe as least impactful in their transformation toward reform-minded instruction?	Mathematics teachers' perception of the situated apprenticeship professional development program	Panorama Teacher Survey (Panorama Education, 2015) Semi-structured interviews	Descriptive Statistics A priori coding Emergent coding
To what extend was the situated apprenticeship professional development program implemented as designed?	Implementation Fidelity	SAPD session audio recordings SAPD session artifacts Researcher's field notes Attendance logs	Descriptive Statistics A priori coding Emergent coding

PAULA L. CLARK

7073 TUNBRIDGE DRIVE, MENTOR, OHIO 44060
PHONE (440) 853•8156 E-MAIL PAULA.CLARK@WESCHOOLS.ORG

EDUCATION

Johns Hopkins University — Baltimore, Maryland
Doctor of Education/Mind, Brain and Teaching — 2019
Dissertation Title:

Cleveland State University — Cleveland, Ohio
7-12 Integrated Mathematics Certificate — 2002

Cleveland State University — Cleveland, Ohio
Master of Education/Curriculum and Instruction — 1997

Hiram College — Hiram, Ohio
Bachelor of Arts in Education/Elementary Education — 1989

Lakeland Community College — Mentor, Ohio
Associate of Arts/ Liberal Education — 1987

LICENSURE

5 Year Professional License – State of Ohio Department of Education

- Integrated Mathematics (7-12)

Permanent License – State of Ohio Department of Education

- Elementary Education (1-8)

WORK EXPERIENCE

2000 – Present Willoughby-Eastlake City Schools — Eastlake, Ohio
Eastlake North High School - *Mathematics Teacher*

- **Instruction**

Currently teach: Advanced Placement Statistics, Honors Algebra II, and Blended Learning (a compilation of Algebra I, Geometry and Algebra II for credit recovery)

Experienced Teaching: Algebra I, Geometry, Honors Geometry, Integrate II

Generates daily lessons. Creates and aligns daily lessons in accordance to district curriculum and Common Core standards for mathematics.

Adapts content to real-world applications. Ensures that students make connections to content learned in the classroom by integrating real-world scenarios.

Creates student-led investigations. Designs lessons that require students to investigate topics, complete projects, and discover mathematical patterns and concepts

- **Leadership**

Professional Development Committee (2014-present) Assists administration in planning and presenting district wide professional development that trains teachers on systemic instructional strategies to promote student growth. Collaborates with colleagues on analyzing such strategies for the improvement of district achievement

Blended Learning Team (2014 – 2016) Assists administration in creating program for at-risk students to allow them to earn high school credits in a non-traditional pathway. Creates curriculum and collaborates with administration and colleagues to implement programs aimed at improving student achievement, attendance and goal setting.

Building Leadership Team (2013 – present) Serves as liaison between the Teacher Based Team and the District Leadership Team on district wide issues regarding curriculum, instruction, and student growth. Participates in and assumes leadership in developing professional development that aligns with district goals.

Ohio Resident Educator Mentor(2006 – present) Serves as mentor to first through third year teachers in Ohio following the Resident Educator Program. Leads new teachers in developing their teaching in order to obtain their professional license in the state of Ohio.

Department Chair of the Mathematics Department (2010 - present). Consults with administration on department issues regarding personnel, curriculum, instruction, facilities, equipment, and supplies within the department. Participates in and assumes leadership, in the development of curriculum guides and materials. Communicates with mathematics department regarding aforementioned issues and relays concerns of department to administration.

PRESENTATIONS

Doctor of Education Residency Program - Johns Hopkins University — 2019
Family and Friends co-presenter

Doctor of Education Residency Program - Johns Hopkins University — 2018
Family and Friends co-presenter

MEMBERSHIP

National Council of Teachers of Mathematics (NCTM) — 2016 - present

Ohio Council of Teachers of Mathematics (OCTM) — 2016 - present